Lecture 2
Advanced Metal Plasticity

ANSYS Mechanical
Advanced Nonlinear Materials
In this Lecture, the advanced rate independent nonlinear kinematic hardening option offered in the Chaboche model is presented.

This plasticity model useful for simulating ratcheting and shakedown.

The following topics will be covered:

A. Review of Linear Kinematic Hardening
B. Nonlinear Kinematic Hardening
C. The Chaboche Model
D. Defining the material input
E. Ratcheting and Shakedown
F. Determining Chaboche Coefficients
A. Review of Linear Kinematic Hardening

For *linear kinematic hardening*, the yield surface *translates* as a rigid body during plastic flow.

- An initially isotropic plastic behavior is no longer isotropic after yielding (kinematic hardening is a form of anisotropic hardening)
- The elastic region is equal to twice the initial yield stress. This is called the Bauschinger effect.
... Review of Linear Kinematic Hardening

The yield criterion can therefore be expressed as:

\[ F = \sqrt{\frac{3}{2}} (s - \alpha) : (s - \alpha) - \sigma_y = 0 \]

where \( s \) is the deviatoric stress, \( \sigma_y \) is the uniaxial yield stress, and \( \alpha \) is the back stress (location of the center of the yield surface).

- Note in the previous plot that the center of the yield surface has translated \( \alpha \). Hence, based on the location \( \alpha \), the yield in reversal is still \( 2\sigma_y \).
- The back stress is *linearly* related to plastic strain via:

\[ \Delta \alpha = \frac{2}{3} C \Delta \varepsilon_{pl} \]
... Review of Linear Kinematic Hardening

Bilinear kinematic hardening (BKIN) is an example of linear kinematic hardening.

• As explained on the previous slide, the term “linear” refers to the relationship of back stress and equivalent plastic strain.

Can be used for cyclic loading since it includes the Bauschinger effect (elastic region equal to twice the initial yield stress).

However, linear kinematic hardening is recommended for situations where the strain levels are relatively small (less than 5-10 % true strain).

• Because there is only one plastic slope (tangent modulus), this is not representative of true metals as the hardening is constant.
A note on *multilinear kinematic hardening*:

- Multilinear kinematic hardening is different than BKin, and it uses the Besseling (a.k.a. sublayer or overlay) model. It characterizes multilinear behavior as a series of elasto-perfectly plastic ‘subvolumes,’ each of which yields at different points, so no back stress $a$ is used.
B. Nonlinear Kinematic Hardening

Nonlinear kinematic hardening is similar to linear kinematic hardening except for the fact that the evolution law has a nonlinear term (the “recall” term $g(a)$):

$$\Delta \alpha_i = \frac{2}{3} C_i \Delta \varepsilon_{pl} - \gamma_i \alpha_i \lambda$$

Where $\varepsilon_{pl}$ is equivalent plastic strain while $\lambda$ is accumulated plastic strain.

The yield criterion is expressed as:

$$F = \sqrt{\frac{3}{2}} (s - \alpha) : (s - \alpha) - R = 0$$

Where $R$ is a constant defining the yield stress, similar to linear kinematic hardening.
The yield surface can be described graphically as shown below:

- The current yield surface shifts in principal stress space.
- There is a *limiting yield surface*, as explained in the next slide. In other words, the behavior approaches perfectly plastic, unlike linear kinematic hardening, which does not change slope.
Nonlinear kinematic hardening has the following characteristics:

• Nonlinear kinematic hardening does not have a linear relationship between hardening and plastic strain.

• The nonlinear kinematic hardening term is associated with the translation of yield surface. A non-zero value of $\gamma$ results in a ‘limiting value of $\alpha$.’ This means that, unlike linear kinematic hardening, the yield surface cannot translate forever in principal stress space. The translation is limited within a specific region.

• The constant $R$ (yield stress), which describes the size of the elastic domain, is added on the response. If a limiting value of $\alpha$ exists, then a ‘limiting yield surface’ will also exist.

• Nonlinear kinematic hardening is suitable for large strains and cyclic loading, as it can simulate the Bauschinger effect. It can model ratchetting and shakedown (discussed later).
C. Chaboche Model

The Chaboche Kinematic Hardening model is an example of nonlinear kinematic hardening. As noted previously, the yield function is

\[ F = \sqrt{\frac{3}{2}} (s - \alpha) : (s - \alpha) - R = 0 \]
... Chaboche Model

The back stress $\alpha$ is a superposition of up to five kinematic models:

$$\dot{\alpha} = \sum_{i=1}^{n} \dot{\alpha}_i = \frac{2}{3} \sum_{i=1}^{n} C_i \dot{\varepsilon}_{pl} - \gamma_i \alpha_i \lambda + \frac{1}{C_i} \frac{dC_i}{dT} \dot{T} \alpha_i$$

Where:

$n$ is the number of kinematic models to use

$\alpha$ is the back stress (location of the center of the yield surface).

$C_i$ is constant that is proportional to hardening modulus

$\varepsilon_{pl}$ is equivalent plastic strain

$\gamma_i$ is rate of decrease of hardening modulus

$I$ is accumulated plastic strain.

$T$ is temperature

- Note that if $n=1$ and $g_1=0$, Chaboche will reduce to Bilinear Kinematic (no limiting value for $a_1$).
This figure breaks down the Chaboche model parameters and how they are related to each other:

- $n$ is 3, the number of kinematic models combined together.
- $R$ is the yield stress (constant value)
- Values $\alpha_1 - \alpha_3$ are the back stresses calculated from the previous equation. Constants $C_1 - C_3$ and $\gamma_1 - \gamma_3$ are associated with these values.
- $R$ describes the yield surface whereas $\alpha$ describes the shifting of the center of the yield surface.
- Note that $\gamma_3=0$, so there is no limiting surface for $\alpha$. 

\[ R=160 \]
\[ C_1=80000 \text{, } \gamma_1=2000 \]
\[ C_2=10000 \text{, } \gamma_2=200 \]
\[ C_3=2500 \text{, } \gamma_3=0 \]
D. Defining the material input

To define a Chaboche model, from the Engineering Data Tool Box:

- Expand the Plasticity folder to expose Chaboche Kinematic Hardening.
- RMB on the Chaboche model and choose “Include Property.”

The Chaboche model will then appear in the properties dialogue box for the user to enter appropriate values.
Defining the material input

Up to five kinematic hardening models can be defined under Chaboche via Engineering data GUI.

- If more than five models are necessary, use TB, CHAB and TBDATA, in a command object under the geometry branch for the applicable part(s).
... Defining the material input

As with the other metal plasticity models, Chaboche can also be defined with temperature dependence properties in a tabular format.

Table of Properties Row 26: Chaboche Kinematic Hardening

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Temperature [°F]</td>
<td>Yield Stress (psi)</td>
<td>Material Constant C1 (psi)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>16800</td>
<td>6E+07</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>16000</td>
<td>5.7E+07</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>15000</td>
<td>5E+07</td>
</tr>
</tbody>
</table>

Chart of Properties Row 26: Chaboche Kinematic Hardening

Yield Stress vs Temperature [°F]
E. Ratchetting & Shakedown

*Ratchetting* occurs under a *nonsymmetric* stress-controlled loading

• There is a progressive increase in strain at each cycle.

*Shakedown* occurs under a *nonsymmetric* stress-controlled loading

• There is a progressive stabilization of strain at each cycle.
... Ratchetting & Shakedown

Ratchetting modeled with Chaboche using:
- \( n=1 \)
- \( R=980 \)
- \( C_1=224000 \)
- \( \gamma_1=400 \)

Shakedown modeled with Chaboche using:
- \( n=2 \)
- \( R=980 \)
- \( C_1=224000 \)
- \( C_2=20000 \)
- \( \gamma_1=400 \)
- \( \gamma_2=0 \)

Loading
Controlled Stress
Unsymmetry
Considerations for Ratchetting:

- Ratcheting is the accumulation of plastic strain under an unsymmetric stress-controlled cyclic loading.
- Linear kinematic models cannot capture ratchetting, as shown below.

**Bilinear Kinematic Hardening**

**Multilinear Kinematic Hardening**
Considerations for Ratchetting (cont’d):

- On the other hand, a single nonlinear kinematic model \((n=1)\) for the Chaboche model can capture ratchetting, as shown below.

In the figure on the left, the blue lines indicate a symmetric loading sequence. Note that no ratchetting occurs, and it is a stable cycle.

The red lines indicate a nonsymmetric loading sequence. Because the plastic slopes are different (due to value of back stresses), plastic strains continue to accumulate.
Considerations for Ratchetting (cont’d):

- Ratchetting occurs due to the fact that the initial slope in compression (A-B) is different from the slope in tension (C-D). Since the loading is unsymmetric, C-D is nearer to the ‘limiting yield surface’, so its slope is more asymptotic.
Considerations for Shakedown:

- Shakedown is similar to ratchetting. However, instead of the plastic strain steadily accumulating under nonsymmetric loading, it comes to a standstill in shakedown.
Considerations for Shakedown (cont’d):

One way to model shakedown is with at least two *kinematic models in Chaboche* (n=2). One of the two should have $\gamma=0$.

- One kinematic model will have $\gamma_i \neq 0$, which will provide a ratchetting effect, as shown previously.

- On the other hand, another model will have $\gamma_i=0$ to provide a stabilization effect. Recall that $\gamma_i=0$ is equivalent to bilinear kinematic hardening, so there is no ratchetting.

- Together, the two models will provide ratchetting with stabilization after a certain number of cycles. This is called shakedown.
... Ratchetting & Shakedown

Chaboche vs Linear Kinematic Hardening

- Same model with same nonsymmetric cyclic load

**Chaboche Results**

Stress vs Strain

Plastic Strain (EPPL)

**KINH Results**
F. Determining Chaboche Parameters

MAPDL has a curve fitting routine for reading in cyclic test data and determining Chaboche parameters directly.

Chaboche material parameters; $k$ (yield strength), $C_i$ (hardening modulus) and $\gamma$ (rate of decrease of hardening) can be derived per procedures outlined on 1st and 4th references on last slide.

- Using a series of tension-compression, strain controlled tests, symmetrically loaded at different strain amplitudes.
For each set of test data:

- Determine $k$ approximately from the elastic domain. $k$ is usually half the elastic domain.
- Determine the plastic strain range $\Delta \varepsilon^{pl}$
- Determine the stress range $\Delta \sigma$
- Plotting $(\Delta \sigma/2-k)$ against $(\Delta \varepsilon^{pl}/2)$ to estimate the asymptotic value corresponding to $C/\gamma$
Using an appropriate hyperbolic expression, fit the results to solve for $C_1$ and $\gamma_1$ in relation to $\Delta\sigma/2-k$.

$$\frac{\Delta\sigma}{2} - k = \frac{C_1}{\gamma_1} \tanh\left(\gamma_1 \frac{\Delta \varepsilon^{pl}}{2}\right)$$

Note: Curve fitting procedures often benefit from reasonable initial values. Since $C_1$ is the initial hardening modulus, the slope after the yield stress can be taken as an estimate of $C_1$ and through the relation $C_1/\gamma_1$ determined in previous step, an initial value of $\gamma_1$ can be obtained.
References


Chaboche Nonlinear Kinematic Hardening Model, Sheldon Imaoka, Memo Number STI0805A, May 4, 2008
H. Workshop Exercise

Please refer to your Workshop Supplement:

Workshop 2A: Chaboche