**Abstract:** Mechanical systems definition as multibody systems is a modern way of modeling aiming to the real time simulation of complex product's dynamic behavior. The paper presents aspects regarding the kinematics of the transversal mobile coupling as multibody system. First it is presented the structural scheme of the analyzed mobile coupling. Then are defined: the parts of the multibody system associated to the mobile coupling; the body reference frames and also the general reference frame; the geometrical and kinematical constraints. Finally, there are determined the kinematical equations, useful to found the optimal geometrical configuration of the coupling, depending by the initial requests of a mobile coupling and to obtain some new constructive variants, presented in the final part of the paper.

Key words: Mobile coupling, multibody system, coupling kinematics, geometrical and kinematical constraints.

**1. INTRODUCTION**

The transversal couplings are used in torque transmission between two shafts with parallel axis [1]. The shafts 1 and 3 are connected by a planar kinematical linkage 2, which give the possibility to have translations in the transversal plane between the shaft axes. These translations are named transversal movements (fig. 1).

A transversal coupling is homogametic if the shafts angular speeds are identical. If the shafts angular speeds are almost identical, the mobile coupling is quasihomokinetic [1, 2].

The transversal coupling multibody model has, as bodies, the input and output shafts and also, some of the intermediary bodies which have, usually, more than two connections [3].

![Fig. 1. Example of a mobile transversal coupling, with transversal movements.](image1)

The most simple known linkages [1, 2] that may realise a translation between two elements are the parallelogram linkage contour and also the anti-parallelogram linkage contour (fig. 2, where 1, 2, 3, 4 are the links of the linkages).

![Fig. 2. Parallelogram and antiparallelogram linkage contours.](image2)
In practice, is preferable to use the parallelogram linkage contours, which led to homokinetic couplings \([1, 2]\). The anti-parallelogram linkage contours are quasi-homokinetic (in fig. 3, for link 4 it appears the supplementary angle \(\Delta \beta\)) and led to quasihomokinetic couplings \([1, 2]\).

If, for the coupling presented in figure 1, the translation joints are both replaced with parallelogram linkages contours, it will be obtained the mobile transversal coupling, presented in figure 3, known and used in practice as Semiflex \([1, 2]\).

Fig. 3. The Semiflex coupling.

2. THE MULTIBODY MODEL

2.1 The structural multibody model

The Semiflex mobile coupling is composed by two semicouplings 1 and 3, and between them there is the intermediary element 2 (figure 4) \([1, 2]\).

Fig. 4. The coupling structural scheme.

If the semicouplings are connected to the basis, will result the associated mechanism \([1, 5]\). The bodies of the multibody system are: input coupling, 1, intermediary body 2, output semicoupling 3 and the basis 0. Between the bodies there are the geometrical restrictions types rotation (R) and rotation-rotation (RR), detailed in table 1.

Table 1. Restrictions

<table>
<thead>
<tr>
<th>Bodies</th>
<th>Geometrical constraints</th>
<th>Joints</th>
<th>Number of restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1</td>
<td>R</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>0 – 3</td>
<td>R</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>1 – 2</td>
<td>RR+RR</td>
<td>CD, GH</td>
<td>2</td>
</tr>
<tr>
<td>2 – 3</td>
<td>RR+RR</td>
<td>EF, IJ</td>
<td>2</td>
</tr>
</tbody>
</table>

The mobility of the associated mechanism multibody system is \([1, 4, 5]\)

\[ M = 3(n_b - 1) - \sum g_c = 3(4 - 1) - 8 = 1, \]  

where \(n_b = 4\) is the number of the associated mechanism bodies and \(\Sigma g_c = 8\) is the number of the geometrical constraints between the bodies.

2.2 Geometrical and kinematical multibody model

For the Semiflex coupling shown in figure 3, the geometrical and kinematical multibody model is presented in figure 5. The interest point’s coordinates are presented in table 2.

Fig. 5. The geometrical and kinematical multibody model.
Table 2. Points coordinates relative to the local coordinate system.

<table>
<thead>
<tr>
<th>Basis 0</th>
<th>Basis 1</th>
<th>Basis 2</th>
<th>Basis 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0(0, 0); D_0(e_x, e_y)$</td>
<td>$A_1(r_1x, r_1y); B_1(-r_1x, r_1y); C_1(0, 0)$, where: $r_{1x} = r_1 \cos \alpha_1$ and $r_{1y} = r_1 \sin \alpha_1$</td>
<td>$M_2(r_{2x}', -r_{2y}'); N_2(-r_{2x}', -r_{2y}'); P_2(r_{2x}'', -r_{2y}'')$, where: $r_{2x}' = r_2 \cos \alpha_2'$ and $r_{2y}' = r_2 \sin \alpha_2'$, respectively $r_{2x}'' = r_2 \cos \alpha_2''$ and $r_{2y}'' = r_2 \sin \alpha_2''$</td>
<td>$A_3(-r_3x, r_3y); B_3(-r_3x, -r_3y); D_3(0, 0)$, where: $r_{3x} = r_3 \cos \alpha_3$ and $r_{3y} = r_3 \sin \alpha_3$</td>
</tr>
</tbody>
</table>

2.3 Geometrical and kinematical restrictions

For geometrical restrictions definition, it is necessary to know the points coordinates relative to the fixed coordinates system \cite{3}. The general relation of these coordinates is:

$$\begin{bmatrix} x_{M_i} \\ y_{M_i} \end{bmatrix} = \begin{bmatrix} x_{0i} \\ y_{0i} \end{bmatrix} + \begin{bmatrix} \cos \phi_{i1} - \sin \phi_{i1} \\ \sin \phi_{i1} \cos \phi_{i1} \end{bmatrix} r_{xi},$$

where $i=1, 2, 3$.

Generally, for the rotation geometrical restriction (R) between two bodies, the equation is \cite{3, 4}

$$P_i \equiv P_j.$$  \hfill (3)

Also, for the rotation-rotation (RR) geometrical restriction between two bodies, the equation is \cite{3, 4}

$$(x_{pj} - x_{pi})^2 + (y_{pj} - y_{pi})^2 = d^2.$$  \hfill (4)

The kinematical restriction equation is

$$\rho_1 - f(t) = 0.$$  \hfill (5)

Based on the relations (2)…(5), it is obtained an equation system with 9 equations and 9 unknowns:

$$x_{01}, y_{01}, \phi_1, x_{02}, y_{02}, \phi_2, x_{03}, y_{03}, \phi_3.$$

3. THE SOLUTIONS OF THE EQUATIONS SYSTEM

To resolve the equation system, it is necessary to take account by the mobile transversal coupling constructive conditions \cite{1, 2}, as follow:

- the length links between the bodies 1 and 2 and respectively 2 and 3 are equal, $l_1' = l_1'' = l_1$, respectively $l_2' = l_2'' = l_2$; usually, also $l_1 = l_2$;
- at the homokinetic couplings of this type, the construction of semicouplings is
symmetric: \( r_1 = r_1' \), respectively \( r_3 = r_3'' \), and 
\( \alpha_1 = \alpha_2' \), respectively \( \alpha_2 = \alpha_3 \); usually, 
also \( r_1 = r_1' = r_2'' = r_3 \) and \( \alpha_1 = \alpha_2' = \alpha_2'' = \alpha_3 \).

Taking account of previous conditions, and for \( r_1, r_1' \) with arbitrary values, the 
equation system led to:

\[
\begin{align*}
    r_2' \cos \alpha_1' \cos \phi_2 &= r_1 \cos \alpha_1 \cos \phi_1; \\
    r_2' \cos \alpha_1' \sin \phi_2 &= r_1 \cos \alpha_1 \sin \phi_1; \\
    \sin(\phi_1 - \phi_2) &= 0. 
\end{align*}
\]  
(6)

If \( r_2' \cos \alpha_1' = r_1 \cos \alpha_1 \) (in fact \( r_2' = r_1 \), a 
parallelism condition between the links), the result is \( \phi_1 = \phi_2 + 2K\pi \), where 
\( K \in \mathbb{Z} \), for \( \forall \phi_1, \phi_2 \in \mathbb{R} \). For the real case 
\( K = 0 \), it is obtained the homokinetism 
condition between the input semicoupling 
1 and intermediary body 2

\[ \phi_1 = \phi_2. \]  
(7)

Also, for \( r_3, r_3'' \) with arbitrary values, the 
equation system led to:

\[
\begin{align*}
    r_2'' \cos \alpha_3' \cos \phi_2 &= r_3 \cos \alpha_3 \cos \phi_3; \\
    r_2'' \cos \alpha_3' \sin \phi_2 &= r_3 \cos \alpha_3 \sin \phi_3; \\
    \sin(\phi_2 - \phi_3) &= 0. 
\end{align*}
\]  
(8)

If \( r_2'' \cos \alpha_3' = r_3 \cos \alpha_3 \) (in fact \( r_2'' = r_3 \), a 
parallelism condition between the links), the result is \( \phi_2 = \phi_3 + 2K\pi \), where 
\( K \in \mathbb{Z} \), for \( \forall \phi_2, \phi_3 \in \mathbb{R} \). For the real case 
\( K = 0 \), it is obtained the homokinetism 
condition between the intermediary body 2 
and output semicoupling 3

\[ \phi_2 = \phi_3. \]  
(9)

Taking account the relations (7) and (9), it 
is obtained

\[ \phi_1 = \phi_3. \]  
(10)

The relation (18) is the homokinetism 
condition of the Semiflex mobile coupling.

3. THE PARTICULAR CASES

Previously, it was obtained the 
homokinetism condition of the Semiflex 
mobile coupling. This condition is 
available only if the follow equalities are respected:

\[ r_1' \cos \alpha_1' = r_1 \cos \alpha_1 \]  
(11)

and

\[ r_3'' \cos \alpha_3'' = r_3 \cos \alpha_3 \]  
(12)

So, it is necessary to discuss these 
equalities. If we consider \( \alpha_1 \) and \( r_1 \) 
respectively \( \alpha_3 \) and \( r_3 \) known, it is 
necessary to obtain \( \alpha_2' \) and \( r_2' \), respectively \( \alpha_2'' \) and \( r_2'' \). It is obtained:

\[ \cos \alpha_2' = \frac{r_1}{r_2} \cos \alpha_1, \]  
(13)

for \( \alpha_2' \in \mathbb{R} \) and \( \frac{r_1}{r_2} \cos \alpha_1 \in \mathbb{R} \), respectively

\[ \cos \alpha_2'' = \frac{r_3}{r_2'} \cos \alpha_3, \]  
(14)

for \( \alpha_2'' \in \mathbb{R} \) and \( \frac{r_3}{r_2'} \cos \alpha_3 \in \mathbb{R} \).

Taking account of the cosinus function 
properties, will result the possible 
thetical cases presented as follow. For every 
coupling known and used in practice 
\([1, 2, 5]\) there are presented the structural 
and constructive scheme. For every new 
thetical coupling variant it is presented 
just the structural scheme.

First there are presented Semiflex and 
Krawtschenko (fig. 3 and 4, respectively 
6), homokinetic couplings, with the 
particularities:

\[ \alpha_2' = \alpha_1 + 2K\pi, \] 
\[ \alpha_2'' = \alpha_3 + 2K\pi, \]

for \( r_1 = r_1', \ r_3 = r_3'' \).
Then, it is presented Schmidt, homokinetic coupling (fig. 7), with the particularities:

\[
\begin{align*}
\alpha'_1 &= -\alpha_1 + 2K\pi \\
\alpha''_1 &= -\alpha_3 + 2K\pi \\
\text{for } &\quad r_1 = r'_2; \quad r_3 = r''_2.
\end{align*}
\]

Another particular case is Kärger, homokinetic coupling (fig. 8), with the particularities:

\[
\begin{align*}
\alpha'_2 &= 0 \\
\alpha''_2 &= 0 \\
\text{for } &\quad r''_2 < r_3; \quad r'_2 < r_1.
\end{align*}
\]

Next particular case is John, homokinetic coupling (fig. 9), with the particularities:

\[
\begin{align*}
\alpha'_3 &= \pm \arccos\left(\frac{r_1}{r_2\cos \alpha_1}\right) + 2K\pi \\
\alpha''_3 &= \pm \arccos\left(\frac{r_1}{r_2'\cos \alpha_3}\right) + 2K\pi \\
\text{for } &\quad \frac{r'_2}{r_1} > 1; \quad \frac{r''_2}{r_3} > 1.
\end{align*}
\]

Another particular case correspond to a new constructive variant (homokinetic, fig. 10), with the particularities:
$$\alpha'_2 = \arccos\left(\frac{r_1}{r'_2}\right) + 2K\pi$$

$$\alpha''_2 = \arccos\left(\frac{r_1}{r''_2}\right) + 2K\pi$$

for \(\frac{r_1}{r'_2} < 1, \frac{r_1}{r''_2} < 1\) and \(\alpha_1 = 0, \alpha_3 = 0\).

4. CONCLUSION

A mobile coupling as multibody system with three bodies is homokinetic if condition (10) is satisfied. Specifying the coupling constructive conditions is important here, because led to the symmetry, useful in coupling’s dynamic behavior and in manufacturing process [1, 2].

Equalities (11) and (12) led to some particular cases of known and used mobile couplings [1, 2, 5]: Schmidt, Semiflex, Krawtschenko, Kärger, John 7, and also to another three new constructive solutions.

The method can be applied also for other known mobile couplings, as multibody systems with four or five mobile bodies and for other new solutions of mobile couplings, identified previously by the authors in structural analysis with graphs [5].

In the future researches, the authors intend to analyse these new particular cases and also to find their optimal shape configuration, in the design process.

5. REFERENCES