INVESTIGATION OF VIBRATOR WITH AIR FLOW EXCITATION

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Abstract: In the daily life and techniques people all time have interaction with continue media like air. Very large achievements in mechanics of the field of aerodynamics (constructions of aircraft, jet engines and space apparatus) exist. In the same time there are many technical tasks to be solved. One of the tasks group is optimization of form and parameters of technical objects with a criteria of energy saving or its useful utilization from interaction with airflow. In this report motion of vibrator with constant air flow excitation is investigated.

Key words: motion control, air excitation, optimisation, adaptive control, synthesis.

1. INTRODUCTION

In this report a motion of a vibrator only with constant air flow \( V_0 \) excitation in the first approximation is investigated. The main idea is to find out optimal control law for variation of additional area \( S(t) \) of vibrating object within limits (1.):

\[
S_1 \leq S(t) \leq S_2,
\]

where \( V_0 \) - constant velocity of air flow, \( S_1 \) - lower level of area; \( S_2 \) - upper level of area, \( t \) - time. The criterion of optimization is time \( T \) required to move object from initial position to end position.

2. SOLUTION OF OPTIMAL CONTROL PROBLEM

The differential equation of motion is (2) \[^{[1]}\]:

\[
m \ddot{x} = -c \dot{x} - b \dot{x} - u(t) \cdot (V_0 + \dot{x})^2, \quad (2)
\]

\[
u(t) = S(t) \cdot k
\]

where \( m \) – mass, \( \ddot{x} \) - acceleration, \( x \) – displacement of object, \( \dot{x} \) - velocity of object, \( c \) – stiffness of spring, \( b \) – damping coefficient, \( V_0 \) - constant velocity of wind, \( S(t) \) – area variation, \( u(t) \) - control action (3), \( k \) – constant. It is required to determine the control action \( u = u(t) \) for displacement of a system with one degree of freedom (2) from initial position \( x(t_0) \) to end position \( x(T) \) in minimal time \( K = T \) if area \( S(t) \) has limit (1).

For solution of problem the maximum principle of Pontryagin may be used \[^{[2]}\]. We have the high-speed problem.

\[
K = \int_{t_0}^{t_1} \cdot dt.
\]

To assume \( t_0 = 0; \ t_1 = T \), we have \( K = T \).

From system (2), transform

\[
x_1 = x; \quad \dot{x}_1 = x_2 \quad \text{or}
\]

\[
\dot{x}_1 = x_2; \quad m \ddot{x}_2 = -c \dot{x}_2 - b \dot{x}_2 - u(t) \cdot (V_0 + \dot{x})^2
\]

we have Hamiltonian (3):

\[
H = \psi_0 + \psi_1 \cdot x_2 + \\
+ \psi_2 \left( \frac{1}{m} \cdot \left(-cx_1 - bx_2 - u(t) \cdot (V_0 + x_2)^2 \right) \right) \quad (3)
\]
here \( H = \psi \cdot X \), where (4)

\[
\psi = \begin{cases} \psi_0 \\ \psi_1 \\ \psi_2 \end{cases}; \quad X = \begin{cases} 0 \\ \dot{x}_1 \\ \dot{x}_2 \end{cases}.
\] (4)

Scalar multiplication of two last vector functions \( \psi \) and \( X \) in any time (function \( H \)) must be maximal. To have such maximum, control action \( u(t) \) must be within limits \( u(t) = u1; \ u(t) = u2 \), depending only from the sign of function \( \psi_2 \) (5):

\[
H = \max H, \quad \text{if} \quad \psi_2 \cdot (-u(t) - (V_0 + x_2)^2) = \max.
\] (5)

Therefore if \( \psi_2 > 0 \), the \( u(t) = u1 \) and if \( \psi_2 < 0 \), the \( u(t) = u2 \), where \( u1 = S1 \cdot k \) and \( u2 = S2 \cdot k \), see (1). Examples of control action with one, two and three switch points are shown in Fig. 1. – Fig. 3.

For realizing optimal control actions (in general case) system of one degree of freedom needs a feedback system with two adapters: one for displacement measurement and another – for velocity measurement. There is a simple case of control existing with only one adapter when motion changes directions, as shown in Fig. 3. [2]. It means that control action is similar to negative dry friction and switch points are along zero velocity line. In that case equation of motion is (6):

\[
m \cdot \ddot{x} = -c \cdot x - b \cdot \dot{x} - F(t) - \\
+ \left[ k \cdot (V0 + \dot{x})^2 \cdot S1 \cdot (0,5 - 0,5 \cdot \frac{\dot{x}}{|\dot{x}|}) \right] - \\
+ \left[ k \cdot (V0 + \dot{x})^2 \cdot S2 \cdot (0,5 + \frac{\dot{x}}{|\dot{x}|}) \right],
\] (6)

where \( m \) – mass; \( c, b, Ft, k, V0 \) – constants. Examples of modelling are shown in Fig. 4. – Fig. 8.
An attempt to find more, than one limit cycle was investigated for a very complicated system with cubic resistance force and dry friction. Answer is positive: for a system with non-periodical excitation (e.g. constant velocity air flow) there can be more, than one limit cycles (Fig. 9., Fig. 10.).

4. SYNTHESIS OF SYSTEM WITH ROTATING BLADES

Scheme of system include main mass of an object, spring and usual damper (Fig. 11.). Part of mass with blades does not rotate. Another part of main mass such as blades rotates around fixed axis inside body. Blades have symmetric area A1 and A2.
Results of investigation are shown in Fig. 12. – Fig. 15.

Fig. 11. Area variation function in time domain when blades rotate with constant angular velocity $\omega$.

Investigation shows that system is very stable because air excitation and damping forces depend from velocity in second degree.

![Area variation function](image)

Fig. 12. Motion in phase plane for parameters close to first resonance.

5. SYNTHESIS OF SYSTEM INSIDE TUBE WITH VARIABLE HOLE

Adaptive control was analyzed by following area variation function $f_1(A)$ (Fig. 13. – Fig. 14.):

$$f_1(A) = A_2 \cdot (1 + a \cdot \frac{v \cdot x}{|v \cdot x|})$$

![Synthesis of a system inside tube](image)

Fig. 13. Synthesis of a system inside tube

It is shown that adaptive systems are very stable because air excitation and damping forces depends from velocity in second degree. At the end of investigations some experimental works inside wind tunnel are analyzed. Parameters of subsonic wind tunnel were following: Length: 2.98m. Width: 0.8m. Height: 1.83m. Variable speed motor driven unit downstream the working section permits continuous control of airspeed between 0 and 26 ms$^{-1}$. Experiment confirm that airflow excitation is very efficient.

6. CONCLUSION

Is shown that airflow may be used for excitation objects in vibration technique. Control of object area allows us to find very efficient excitation systems. Use of new vibration systems with airflow is in starting position and needs more fundamental investigations.

7. REFERENCES