VIRTUAL REALITY METHOD SUPPORTED SYNTHESIS OF A PLANE MANIPULATOR

Heinloo, M. & Leola, T.

Abstract: This paper deals with the application of virtual reality method in the environment of the computer package Mathcad to the synthesis of a plane manipulator. Prescribed motion of the manipulator is obtained by application the method of superposition of motions to the manipulator of previously optimized dimensions. The process of synthesis is accompanied by thorough current control of the prescribed restrictions by using the graphics features of the computer package Mathcad.

Keywords: Machinery, manipulator, engineering design, virtual reality, computer graphics.

1. INTRODUCTION

A computer-generated simulation of reality (virtual reality) is an essential aid to study and solve the practical problems of machinery. World practice shows that the use of this method to study of a machine element and its working process is often much cheaper and more effective than only a practical experimentation. It is reasonable to start engineering design and experiments only after full application of the method of virtual reality.

Heinloo et al. [1] have used the method of virtual reality for analysis a planar linkage. Heinloo et al. [2] applied this method to the synthesis a manipulator for a scraper of a press manure removal. Heinloo and Olt [3] have used the method of virtual reality in the process of creation a disk–ridging tool.

This paper presents the application a method of virtual reality in the process of synthesis a plane manipulator with specified motion.

2. PRESCRIBED RESTRICTIONS TO THE MOTION OF THE MANIPULATOR

Let us consider a manipulator

and suppose that the prescribed restrictions to its motion are:

- due to the tolerance at the pivot B the link OACED begins the motion at the moment of the time \( t_0 = L/v_r = 0.0263 \) s \( (L = 0.006 \text{ m} – \text{tolerance caused free stroke of the slider B before moving the link OACED; } v_r = 0.2285 \text{ m/s – velocity of the slider B in the moving to the negative direction of y-axis}) \), when the points C, E and D are at the initial position on the \( x_1 \)-axis (Fig. 1);
- point D of the link OACED moves along x-axis and reach at the moment of the time \( t_1 = h/v_r = 0.3632 \) s \( (h = 0.0830 \text{ m – full stroke of the slider B in the local system Oxy}) \) the origin \( O_1 \) and begins then moving along y-axis to its negative direction;

Fig. 1. The scheme of a manipulator
• pivots O and B (sliders) are allowed to move only along their tracks;
• pivot B reach the final position at the moment of the time \( t_2 = 6 \) s;
• due to the tolerance at the pivot B the link OACED begins the turning from the final position to the initial orientation at the moment of the local time \( t'_0 = \frac{L}{v_l} = 0.0431 \) s \( (v_l = 0.1393 \) m/s – the velocity of the slider B in the moving to the positive direction of y-axis of the system Oxy), counted from the beginning of turning the link OACED from final position, when the point D is on the y-axis.
• the x-co-ordinate of the pivot B is fixed \( x_B(t) = \delta \) (1)
• the y-co-ordinate changes in the local system of co-ordinates Oxy according to the law
  \[
y_B(t) = y_B(0) - v_y(t - t_0), (t \geq t_0 > 0), (2)
\]
  when the slider B is moving in this system to the negative direction of y-axis and according to the law
  \[
y'_B(t') = y_B(t) + L + v_y(t' - t'_0), (3)
\]
  when the slider B is moving to the positive direction of y-axis. Here \( t \) – the current time; \( t' \) – the local current time, counted from the beginning the moving of link OACED from the final position \( y_B(0) \) – fixed y-co-ordinate of the pivot B directly before moving the link OACED at the moment of time \( t_0 \), \( \delta \) – the distance of the track of the slider B from y-axis (Fig. 1);
• At the interval \( t'_1 - t'_0 \) of the local times the slider O have to be immovable, the link OACED turns around the pivot O and at the moment of the local time \( t'_1 \) CED takes the initial orientation;
• the points C, E, D are returning to the initial position on \( x_1 \) axis at the moment of the local time \( t'_2 \).

Prescribed here motion can be realized by two drivers (Fig. 2)

- Local driver
- Global driver

Fig. 2. A manipulator with two drivers

3. A VIRTUAL MANIPULATOR

Co-ordinates of the points C, E, D and pivots A, B are determined in the local system of co-ordinates Oxy by the following system of equations:

\[
\begin{align*}
 x_A^2 + y_A^2 &= \rho_{AO}^2, \quad x_E^2 + y_E^2 = \rho_{OE}^2, \\
 (x_A - \delta)^2 + [y_A - y_B(t)]^2 &= \rho_{AB}^2, \\
 (x_E - x_c)^2 + (y_E - y_A)^2 &= \rho_{AE}^2, \\
 x_C^2 + y_C^2 &= \rho_{OC}^2, \\
 (x_c - x_E)^2 + (y_c - y_E)^2 &= \rho_{EC}^2, \\
 \end{align*}
\]

(4)

A possible analytical solution of the system (4) had been derived by Heinloo et al. \[^{[2]}\]. The vectorfunctions

\[
\begin{bmatrix}
 x_A(t) \\
 x_E(t) \\
 x_c(t) \\
 x_A(t) \\
 x_E(t) \\
 x_c(t) \\
 x_A(t) \\
 x_E(t)
\end{bmatrix}
\begin{bmatrix}
 y_A(t) \\
 y_E(t) \\
 y_C(t) \\
 y_A(t) \\
 y_E(t) \\
 y_C(t) \\
 y_A(t) \\
 y_E(t)
\end{bmatrix}
\]

imagine at a moment of the time \( t \) in the local system of co-ordinates Oxy on the worksheet of computer package Mathcad an image of manipulator, named below as virtual manipulator (Fig. 3).
4. ON OPTIMAL POSITIONS

Heinloo et al. [2] considered the manipulator in Fig. 1 with the following parameters:

- \( \rho_{AB} = 0.0837 \) m,
- \( \rho_{DE} = 0.0490 \) m,
- \( \rho_{CA} = 0.0270 \) m,
- \( \rho_{EC} = 0.0560 \) m,
- \( \rho_{DC} = 0.4110 \) m,
- \( \rho_{AE} = 0.0622 \) m,
- \( \rho_{OC} = 0.0718 \) m,
- \( \rho_{AO} = 0.0564 \) m,

and derived values of parameters \( \delta \), \( \rho_{AO} \) as solution of the system of equations

\[
\begin{align*}
\rho_{AO} - \delta &= x_D(t_1, \delta, \rho_{AO}) = 0, \\
\rho_{AO} - \delta &= y_D(t_0, \delta, \rho_{AO}) = y_C(t_0, \delta, \rho_{AO}).
\end{align*}
\]

This system has the solution \( \delta = 0.0177 \) m, \( \rho_{AO} = 0.0564 \) m. Fig. 4 shows the optimal positions of the virtual manipulator in the local co-ordinate system Oxy.

5. SUPERPOSITION OF MOTIONS

Required motion of the point D along the global axis O_{1X_1} (Fig. 1) at the interval of the time \( t_0 \leq t \leq t_1 \) can be obtained by the following superposition of motions:

\[
S_x(t) = s_x(t), \quad S_y(t) = s_y(t) - y_D(t). \tag{5}
\]

Fig. 5 shows optimal positions of the virtual manipulator in the global system O_{1X_1Y} of co-ordinates.
It follows directly from Fig. 5 that prescribed restriction to the motion of the point D (Fig. 1) of the virtual manipulator is satisfied in the interval $t_0 \leq t \leq t_1$ of the time $t$.

6. FIRST TRAVERSE OF THE VIRTUAL MANIPULATOR

According to the prescribed requirements, the local co-ordinate system Oxy together with the virtual manipulator (Fig. 1) has to traverse negative direction of the $y$–axis at the interval $t_1 \leq t \leq t_2$ of the time $t$. Let the traversing velocity in this interval determines the formula

$$v_{\alpha}(t) = 0.2 \left\{ 1 - \cos \left[ \frac{2\pi (t - t_1)}{t_2 - t_1} \right] \right\}.$$

Fig. 6. The dependence of the traversing velocity of the slider O on the time $t$.

Fig. 6 shows the traversing velocity $v_O(t)$ of the slider O at the interval $t_0 \leq t \leq t_2$ of the time $t$. The traversing of the manipulator at the interval $t_1 \leq t \leq t_2$ of the time $t$ can be determined by the functions

$$T_y(t) = s_y(t) - \int_{t_1}^{t} v_{\alpha}(t) dt, \quad T_x(t) = s_x(t).$$

As above, let us define the vectors

$$b_x = \begin{cases} u \leftarrow T_x(t_1) \\ \text{for } t \in t_1, t_1 + \frac{t_2 - t_1}{15} \ldots t_2 \\ u \leftarrow \text{stack}(u, T_x(t)) \end{cases}$$

$$b_y = \begin{cases} u \leftarrow T_y(t_1) \\ \text{for } t \in t_1, t_1 + \frac{t_2 - t_1}{15} \ldots t_2 \\ u \leftarrow \text{stack}(u, T_y(t)) \end{cases}$$

The vectors $b_x$ and $b_y$ allow to image the first traversing (Fig. 7) of the manipulator to the negative direction of the $y$–axis (Fig. 1) at the interval $t_1 \leq t \leq t_2$ of the time $t$.

7. TURNING THE LINK OACED

Let us consider firstly the turning of the manipulator at the interval $t'_0 \leq t' \leq t'_1$ of the local time $t'$, counted from the beginning of motion the slider B, in the local system of co–ordinates Oxy. In this interval the slider B moves in the system Oxy according to the law (3). Because of tolerance in the pivot B the length $\rho'_{AB}$ of the link AB have to be determined by the formula

$$\rho'_{AB} = \sqrt{\left[ y_b(t') + L - y_a(t') \right]^2 + [\delta - x_a(t')]^2}.$$

Co-ordinates of the points C, E, D and pivots A, B in the local co–ordinate system determines the system of equations

$$x_A'^2 + y_A'^2 = \rho_{AO}^2,$$

$$\left( x_A' - \delta \right)^2 + y_A'^2 + y_b(t)^2 = \rho'_{AB}^2,$$

$$x_E'^2 + y_E'^2 = \rho_{OE}^2.$$
\[
\begin{align*}
(x_E - x_A^*)^2 + (y_E - y_A^*)^2 &= \rho_{ae}^2, \\
x_C^2 + y_C^2 &= \rho_{oc}^2, \\
(x_C^* - x_E^*)^2 + (y_C^* - y_E^*)^2 &= \rho_{ec}^2, \\
x_D^* &= \frac{\rho_{dc}}{\rho_{ec}} x_C^* + \left(\frac{\rho_{ec} - \rho_{dc}}{\rho_{dc}}\right) x_C^*, \\
y_D^* &= \frac{\rho_{dc}}{\rho_{ec}} y_C^* + \left(\frac{\rho_{ec} - \rho_{dc}}{\rho_{dc}}\right) y_C^*
\end{align*}
\]  
(5)

Local time \( t_1 \) can be determined from the equation \( y_D(t_1) = y_C(t_1) \). For considered here parameters \( t_1 = 0.6002 \) s.

The virtual manipulator returns to the initial position at the time \( t_2 \), which can be determined from the equation

\[
\int_0^{t_2} v_y(t) \, dt + y_D(t_2) - y_D(t_0) = \int_0^{t_2} v_y(t', t_2) \, dt,
\]

where

\[
v_y(t', t_2) = 0.1 \left[ 1 - \cos \left( \frac{2\pi(t - t_1)}{t_2 - t_1} \right) \right]
\]
is the second traversing velocity. For considered here parameters \( t_2 = 15.015 \) s.

Let us compose the following vector-functions:

\[
s_x(t) = \begin{bmatrix}
x_O^* (t) \\ x_E (t) \\ x_D^* (t) \\ x_B (t) \\ x_A^* (t) \\ x_O^* (t)
\end{bmatrix},
\]

\[
s_y(t) = \begin{bmatrix}
y_O^* (t) \\ y_E (t) \\ y_D^* (t) \\ y_B (t) \\ y_A^* (t) \\ y_O^* (t)
\end{bmatrix}
\]

\[
W_x(t') = s_x(t') - \int_{t_1}^{t_2} v_x(t', t_2) \, dt - y_D(t_0),
\]

\[
W_y(t') = s_y(t') - \int_{t_1}^{t_2} v_y(t', t_2) \, dt - y_D(t_0)
\]

Vectors

\[
c_x := u \leftarrow W_x(0 \cdot s)
\]

for \( t \in 0, \frac{t_1}{30} \ldots t_1 \)

\[
\]

\[
c_y := u \leftarrow W_y(0 \cdot s)
\]

for \( t \in 0, \frac{t_1}{30} \ldots t_1 \)

are image the turning of the link OACED from final traversing position to the initial orientation (Fig. 8).

8. SECOND TRAVERSE OF THE VIRTUAL MANIPULATOR

To traverse the virtual manipulator from the final position with initial orientation to the initial position let us define the functions

\[
W_x(t) = s_x(t_1),
\]

\[
W_y(t) = s_y(t_1) - \int_{t_1}^{t_2} v_y(t', t_2) \, dt - y_D(t_0)
\]

and vectors
\[ d_x := \begin{align*}
&= \begin{cases} 
0 & \text{for } t \in t_1', t_1' + \frac{r_2' - r_1'}{30} \ldots t_2' \\
&
\end{cases} \\
&= \begin{cases} 
0 & \text{for } t \in t_1', t_1' + \frac{r_2 - r_1}{30} \ldots t_2' \\
&
\end{cases}
\end{align*} \\
&= \begin{cases} 
0 & \text{for } t \in t_1', t_1' + \frac{r_2 - r_1}{30} \ldots t_2' \\
&
\end{cases} \\
&= \begin{cases} 
0 & \text{for } t \in t_1', t_1' + \frac{r_2 - r_1}{30} \ldots t_2' \\
&
\end{cases}
\]

Vectors \( d_x \) and \( d_y \) image the transverse of the virtual manipulator to the initial position.

Fig. 9. The Traverse of the virtual manipulator back to the initial position.

To image the full cycle of motion the special video clip has been made. This clip shows, that all prescribed restrictions are satisfied exactly.

9. CONCLUSIONS

The method of virtual reality used in this paper for synthesis a manipulator, which motion satisfies prescribed restrictions can be easily generalized to the design of various machine elements. The images of positions of mechanism and the video clips allow thorough current control the satisfaction of prescribed restrictions. Application of virtual reality methods to the process of engineering design and studing/understanding the working process of machine element is important.

10. ADDITIONAL DATA ABOUT AUTHORS

Mati Heinloo, 
Prof., Dr. 
Estonian University of Life Sciences, 
Kreutzwaldi 56, 51014 Tartu, Estonia. 
Mati.Heinloo@emu.ee 
Phone: +372 55 10 512 
Fax: +372 731 3334

Taavi Leola 
M.Sc., Ph.D Student, 
Estonian University of Life Sciences, 
Kreutzwaldi 56, 51014 Tartu, Estonia. 
ttaavi@emu.ee 
Phone: +372 56 477 439 
Fax: +372 731 3334

Corresponding author: Mati Heinloo

11. REFERENCES


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