

## DESIGN OF A RADIAL CAM FOR THE CAM-FOLLOWER MECHANISM

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**Abstract:** This paper presents a method for designing of a radial cam on the worksheet of Mathcad including the calculation of follower's displacements, values of its velocities and accelerations, the optimization of the contour of a cam by choosing the eccentricity of follower and the radius of base circle of cam and the simulation of the working process of the cam mechanism. The optimal coordinates the contour of the cam were calculated. Finally, practical information for exporting cam contour data from Mathcad environment in data format necessary for CNC part program generation is discussed, considering CNC controllers using the most widespread ISO type program format and contouring with linear interpolation.

**Key words:** cam mechanism, design, optimization, visualization.

### 1. INTRODUCTION

Cam mechanisms consisting of only two elements, the cam as mechanical driver element and the follower as driven element, are widely used in technical applications due to reduced constructive complexity. Any functional motion of follower due to the movement of cam can be produced [1].

Cam mechanism can be classified in several ways: by follower motion (translating, rotating), by type of cam (radial, cylindrical), by joint closure, by follower (flat, rolling, mushroom), by cam curves [2].

Alaci et al. [3] have showed some aspects concerning the design of cam mechanisms with oscillating flat faced follower by using computer software. They also have generated figures of cam profiles as enve-

lopes of the straight lines paths on the Mathcad worksheet.

In the study of Ahmet Shala and Ramë Likaj [4] an analytical method for synthesis of cam mechanism is presented. They use Mathcad for displacement, velocity and acceleration calculation of the follower.

This paper presents the method and the results of numerical analysis of the radial cam, with translating follower on the worksheet of Mathcad. The visualization of the cam and follower and the simulation of working process of cam-follower mechanism are presented. The coordinates of the visualized cam contour coordinates can be used in the CNC milling machine for the production of the real cam.

### 2. MATERIALS AND METHODS

#### 2.1 The scheme of cam-follower mechanism

Fig 1 shows the scheme of a cam-follower mechanism

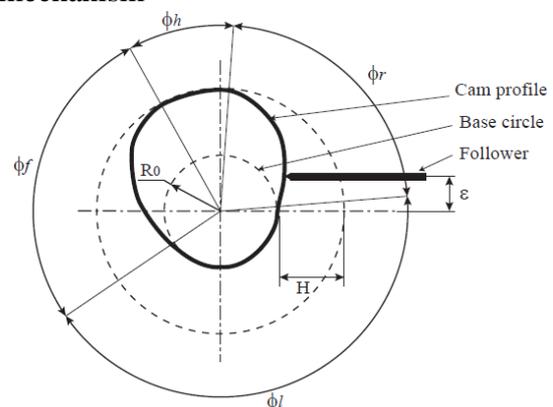


Fig. 1. The scheme cam mechanism, where  $\phi_f$  is the rise angle,  $\phi_h$  – the high dwell angle,  $\phi_f$  – the fall angle,  $\phi_l$  – the low dwell angle,  $\varepsilon$  – the eccentricity,  $H$  – the follower stroke

Let us consider the cam-follower mechanism by the following parameters: the rise angle of the follower  $\phi_r = 60^\circ$ ; the high dwell angle of the follower  $\phi_h = 60^\circ$ ; the fall (return) angle of the follower  $\phi_f = 60^\circ$ ; the stroke of the follower  $H = 0.005m$ ; the offset of the follower  $\varepsilon$ ; the radius of the base circle of the cam  $R_0$ ; the number of positions of cam in computations  $N = 360$ ; angular velocity of the cam  $\omega = 10rad/s$ .

## 2.2 Follower's displacement

According to [2] let us describe the displacement of the follower in the rise by the law:

$$s_1(\varphi) = H \left[ 35 \left( \frac{\varphi}{\phi_r} \right)^4 - 84 \left( \frac{\varphi}{\phi_r} \right)^5 + 70 \left( \frac{\varphi}{\phi_r} \right)^6 - 20 \left( \frac{\varphi}{\phi_r} \right)^7 \right] \quad (1)$$

and in the fall by the law

$$s_2(\varphi) = H - H \left[ 35 \left( \frac{\varphi - \phi_r - \phi_h}{\phi_f} \right)^4 - 84 \left( \frac{\varphi - \phi_r - \phi_h}{\phi_f} \right)^5 + 70 \left( \frac{\varphi - \phi_r - \phi_h}{\phi_f} \right)^6 - 20 \left( \frac{\varphi - \phi_r - \phi_h}{\phi_f} \right)^7 \right]. \quad (2)$$

In (1), (2)  $\varphi$  is the rotation angle of a cam from the initial position.

The displacement of a follower in one cycle of cam's rotation can be determined by the following program on the worksheet of the Mathcad (Fig. 2)

$$s(\varphi) = \begin{cases} s_1(\varphi) & \text{if } 0 \leq \varphi \leq \phi_r \\ s_1(\phi_e) & \text{if } \phi_r < \varphi \leq \phi_r + \phi_h \\ s_2(\varphi) & \text{if } \phi_r + \phi_h < \varphi \leq \phi_r + \phi_h + \phi_f \\ 0 & \text{otherwise} \end{cases}$$

Fig. 2. Program for determination of the follower displacement

Accordingly to program (Fig. 2) we get follower displacement on fig. 3.

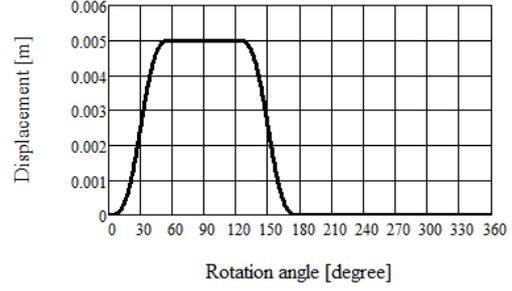


Fig. 3. The dependence of the displacement of the follower on rotation angle  $\varphi$  of the cam

## 2.3 Follower's velocity

The first derivative from equations (1) and (2) gives the velocity analogue functions:

$$v_1(\varphi) = \frac{d}{d\varphi} s_1(\varphi), \quad (3)$$

$$v_2(\varphi) = \frac{d}{d\varphi} s_2(\varphi). \quad (4)$$

The velocity analogue in one cycle of cam's motion can be determined by the program on the figure 4.

$$v(\varphi) = \begin{cases} v_1(\varphi) & \text{if } 0 \leq \varphi \leq \phi_r \\ 0 & \text{if } \phi_r < \varphi \leq \phi_r + \phi_h \\ v_2(\varphi) & \text{if } \phi_r + \phi_h < \varphi \leq \phi_r + \phi_h + \phi_f \\ 0 & \text{otherwise} \end{cases}$$

Fig. 4. Program for determination of the follower velocity analogue

The program in fig. 4 computes the analogue of the follower's velocity in the dependence on the rotation angle  $\varphi$  of the cam. To determine follower's real velocity (fig. 5) the velocity analogue were multiplied by the angular velocity  $\omega$  of the cam.

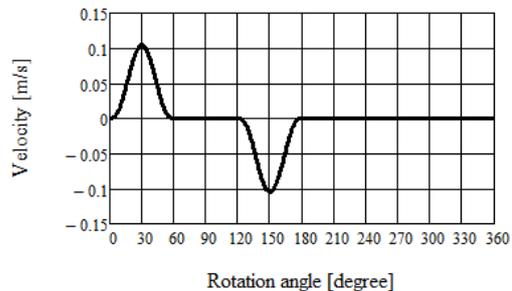


Fig. 5. The dependence of the follower velocity on the rotation angle  $\varphi$  of the cam

## 2.4 Follower's acceleration

The second derivative from equations (1) and (2) gives the acceleration analogue functions:

$$a_1(\varphi) = \frac{d^2}{d\varphi^2} s_1(\varphi), \quad (5)$$

$$a_2(\varphi) = \frac{d^2}{d\varphi^2} s_2(\varphi). \quad (6)$$

The acceleration analogue for one cycle of cam's motion can be determined by the program on the fig. 6.

$$a(\varphi) = \begin{cases} a_1(\varphi) & \text{if } 0 \leq \varphi \leq \phi_r \\ 0 & \text{if } \phi_r < \varphi \leq \phi_r + \phi_h \\ a_2(\varphi) & \text{if } \phi_r + \phi_h < \varphi \leq \phi_r + \phi_h + \phi_f \\ 0 & \text{otherwise} \end{cases}$$

Fig. 6. Program for determination of the follower acceleration analogue

The program in fig. 6 computes the follower's acceleration analogue in the dependence on the rotation angle  $\varphi$  of the cam. To determine follower's acceleration (fig. 7) the acceleration analogue were multiplied by the square of the cam angular velocity  $\omega^2$ .

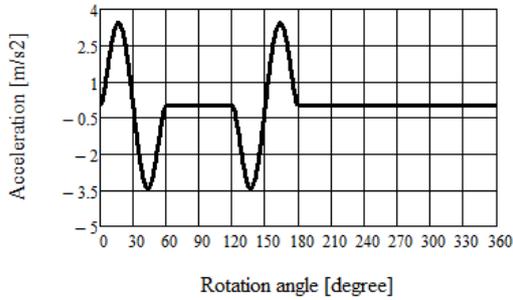


Fig. 7. The dependence of the follower acceleration on the rotation angle  $\varphi$  of the cam

## 2.5 Optimization of the eccentricity and the radius of the base circle

The pressure angle  $\alpha$  (fig. 8) is one of the limiting criteria's for cam design. Accordingly to Norton [2] the pressure angle must be between 0 to  $\pm 30$  degrees. Pressure angle value can be adjusted by the values of the eccentricity  $\varepsilon$  and radius  $R_0$  of base circle.

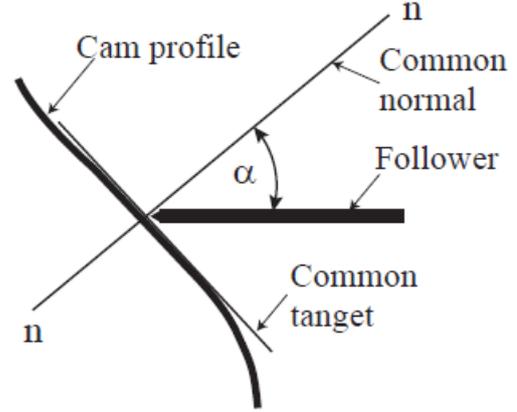


Fig. 8. The pressure angle  $\alpha$  between the cam and the follower

According to [2] the pressure angle  $\alpha$  of the cam determines the following formula:

$$\alpha(\varphi, R_0, \varepsilon) = \arctan \left( \frac{v(\varphi) - \varepsilon}{s(\varphi) + \sqrt{R_0^2 - \varepsilon^2}} \right). \quad (7)$$

Base circle radius and eccentricity cannot be solved conveniently directly. To find proper  $R_0$  and  $\varepsilon$  Norton [2] suggest to use special cam design program Dynacam or some equation solvers such as Matlab, TKSolver or Mathcad.

To determine the radius of base circle  $R_0$  and eccentricity  $\varepsilon$  let us formulate the following problem of optimization:

find such values for  $R_0$  and  $\varepsilon$  that guarantee satisfaction of the restriction

$$\alpha(\varphi, R_0, \varepsilon) = \alpha_e \quad (8)$$

in the rise and the restriction

$$\alpha(\varphi, R_0, \varepsilon) = \alpha_n \quad (9)$$

in the fall.

With lower accuracy the solution of system of equations (8) and (9) can be obtained also by hand [2], changing the values of  $R_0$  and  $\varepsilon$  from initial values step by step. Doing so it is useful to know that the change of value  $\varepsilon$  moves the graph of the function  $\alpha(\varphi, R_0, \varepsilon)$  upwards or downwards without changing its shape and the change of value  $R_0$  increases or decreases the external values of the function  $\alpha(\varphi, R_0, \varepsilon)$ . In such way the solution for system of equations (8) and (9) by low accuracy can be obtained relatively quickly satisfying the need of teaching process.

The solution method of high accuracy for the system of equations (8) and (9) with respect to  $R_0$  and  $\varepsilon$  by the help of Mathcad will be developed in future.

Let us consider an example with the maximal value of the pressure angle  $\alpha(\varphi, R_0, \varepsilon)$   $a_e = 20^\circ$  (in rise) and minimal value of the pressure angle  $a_n = -24^\circ$  (in fall). Let's take the initial values  $R_0 = 20\text{mm}$  and  $\varepsilon = 0\text{mm}$ . In this case the curve of the function  $\alpha(\varphi, R_0, \varepsilon)$  is presented in fig. 9, showing the pressure angle curve crossing the values of  $a_e = 20^\circ$  and  $a_n = -24^\circ$ . This means that the system of equations (8) and (9) is not satisfied.

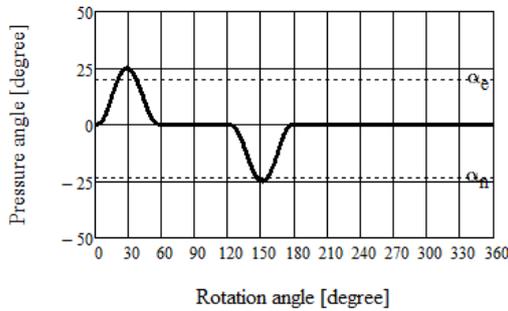


Fig. 9 Cam mechanism pressure angle,  $R_0 = 20\text{mm}$  and  $\varepsilon = 0$

By changing the values of  $R_0$  and  $\varepsilon$  step by step one can visually observe the change of the graph of function  $\alpha(\varphi, R_0, \varepsilon)$  and quickly obtain the position of this graph, shown in fig. 10. This means that the system of equations (8) and (9) is satisfied with some compromise of accuracy. The case in fig. 10 corresponds approximately to  $R_0 = 31\text{mm}$  and  $\varepsilon = -1.5\text{mm}$ .

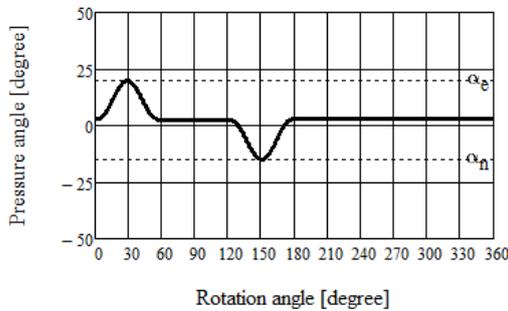


Fig. 10 Cam mechanism pressure angle,  $R_0 = 31\text{mm}$  and  $\varepsilon = -1.5\text{mm}$

## 2.6 Visualization of the cam and simulation of the working process of cam-follower mechanism

The polar coordinates  $\beta(\varphi), R(\varphi)$  of contact point of the follower and the cam are

$$R(\varphi) = \sqrt{s(\varphi)^2 + R_0^2 + 2 \cdot s(\varphi) \sqrt{R_0^2 - \varepsilon^2}}, \quad (10)$$

$$\beta(\varphi) = \varphi - \left( \text{asin} \left( \frac{\varepsilon}{R(\varphi)} \right) - \text{asin} \left( \frac{\varepsilon}{R_0} \right) \right), \quad (11)$$

where  $R(\varphi)$  is the polar radius and  $\beta(\varphi)$  – the polar angle of the contour point in dependence of angle of cam's rotation  $\varphi$ .

To visualize the contour of the cam let us assume that the cam is not rotating but the follower rotates around the centre of the cam with angle of rotation  $\varphi$ . Then the contact point with polar coordinates  $\beta(\varphi), R(\varphi)$  draws the contour of the cam. Fig. 11 visualizes the initial position of the cam in the case  $R_0 = 31\text{mm}$  and  $\varepsilon = -1.5\text{mm}$ .

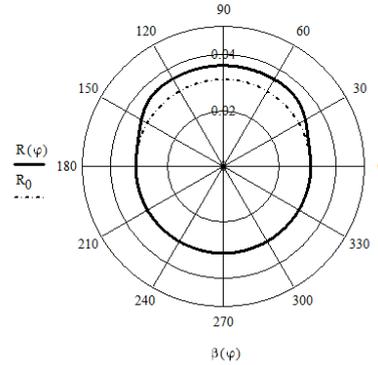


Fig. 11 Cam contour in the polar coordinates  $\beta(\varphi), R(\varphi)$

The Cartesian coordinates of the cam contour at the initial position are

$$x(\varphi) = R(\varphi) \cos(\beta(\varphi)), \quad (12)$$

$$y(\varphi) = R(\varphi) \sin(\beta(\varphi)). \quad (13)$$

To simulate the working process of the cam-follower mechanism the cam in fig. 11 [5] is rotated by the angle  $\theta = \omega t$ , where  $\omega$  is the angular velocity and  $t$  – the time. Equations

$$x_1(\varphi) = x(\varphi) \cdot G(\theta), \quad (14)$$

$$y(\varphi)_1 = y(\varphi) \cdot G(\theta), \quad (15)$$

where

$$G(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

determine the contour of the cam, turned around the centre of cam by the angle  $\theta$  counter clockwise.

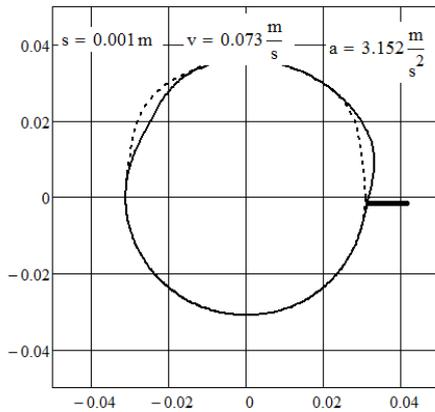


Fig. 12 A frame from video clip [5]

### 3. PRODUCTION OF CAM

Nowadays any part despite of its complicated shape can be machined on modern industrial CNC machine tools with geometrical accuracy of at least 15 microns, so the machining of the modelled geometrical shape of a cam can be accomplished quite easily with great precision. An ordinary radial cam can be produced in CNC machining centre by contour cutting with a square end mill of suitable type according to cam material and dimensions. Cam geometry points in Cartesian coordinates can be calculated with required accuracy according to equations (12) and (13) (fig. 13) by changing the angular increment of calculations.

$x(\varphi) =$		$y(\varphi) =$	
31	mm	0	mm
30.999		0.271	
30.995		0.541	
30.989		0.811	
30.981		1.082	
30.971		1.352	
30.958		1.622	
30.944		1.893	
...		...	

Fig. 13 Contour coordinates of optimal cam

Different geometrical interpolation techniques for connecting the calculated con-

tour points of cam can be available and used in CNC machine tools depending on the machine tool control options. As discussed by Norton [2], circular and spline fitting interpolation techniques are superior to linear interpolation in terms of vibratory noise in acceleration measurements. Still, on most CNC controllers in everyday use only linear or circular tool movement interpolation is available, so the geometrical accuracy of a machined cam (so called “chordal deviation”) compared to mathematical model must be regulated by selecting appropriate angular increment for contour points calculation.

Coordinate values on the fig. 13 can be exported from Mathcad to another different file formats (for example to Microsoft Excel or Notepad) for converting exported data into file format suitable for direct usage in CNC machine tool part program. Majority of CNC part programs are composed in the form of a simple ASCII text file. Widespread CNC controllers using ISO or similar program format accept the input of contour point values for linear interpolation in the form of successive lines of type

```
X...Y... ,
characters X and Y hereby representing the
machine tool axes followed by numerical
coordinate value, for example (numerical
values taken from fig.13)
...
X30.989Y0.811
X30.981Y1.082
X30.971Y1.352 .
```

Coordinate output in showed format can be accomplished from Mathcad calculations.

As the cutting tool centre must be offset from cam contour by the radius value of the cutter, CNC controller’s cutter radius compensation function must be used in process of machining. The optimal method for bringing the tool into cut for closed contours is to approach the contour tangentially, which helps to avoid contouring mistakes caused by tool deflection, so the tangent line direction for the starting point of

contour cutting must be known. Considering the fact the type of cam contour studied in this work is an arc between rotation angles  $\varphi=180\dots360$ , we can assume that the tangent line for contour point  $\varphi=0=360$  is parallel to Y axis (fig. 14). As the machining of contour is started and finished in point  $\varphi=0$ , additional tool moves parallel to X and Y axis can be manually added to program for smooth tool transition into and out of the cut.

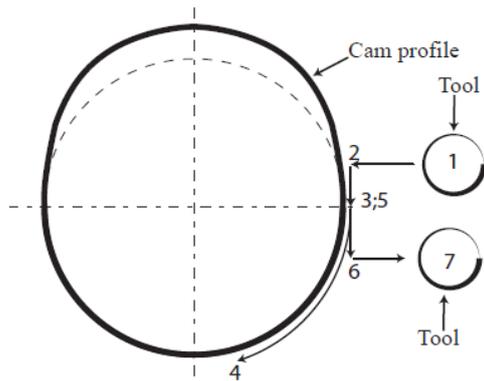


Fig. 14 Tool movement steps: 1 – initial position, 2 – beginning of contour start point tangent line, 3 – starting point of contour cutting, 4 - contour cutting, 5 – end point of contour cutting, 6 – end of contour end point tangent line, 7 – end position

In general the contours are milled using climb cutting, although conventional cutting may give better results in some special cases. The choice between climb and conventional cutting affects the machining direction of a contour, which are accordingly clockwise or counter clockwise for external contours. However, the choice between cutting direction can be easily taken into account while exporting coordinate data from Mathcad. Defining the rotation angle range  $\varphi$  to change clockwise (from 360 to 0 instead) results in contour point coordinates output in clockwise direction and vice versa.

#### 4. CONCLUSION

1. The Computer Package Mathcad can be considered as a convenient tool for design of cams.

2. Mathcad allows the simulation of motion of virtual models of cam mechanisms.  
3. Mathcad can be used to compute cam contour coordinates, which can be used for CNC part program.  
4. The method presented in this paper can be used in the teaching process of engineering subjects and also by engineers in their work.

#### 5. REFERENCES

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