THE EFFECT OF THE SHAPE PARAMETERS ON MODAL PROPERTIES OF ULTRASONIC HORN DESIGN FOR ULTRASONIC ASSISTED MACHINING

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Abstract: In many machining processes, the positive effects of ultrasound are applied. Ultrasonic horn is an important element of the ultrasonic excitation system. The role of horn is the transfer of ultrasonic vibration energy from ultrasonic transducer to the tool cutting edge interacting with the workpiece (ultrasonic assisted machining-UAM). The UAM system performance depends on well-designed ultrasonic horn. The most important aspects of horn design are a horn resonant frequency, amplification factor and the determination of horn resonant wavelength - usually integer multiple of half wavelength. The effect of geometrical parameters of different horn shapes (cylindrical, tapered, exponential) on horn dynamical properties is presented in this paper. The horn modal properties are determined using finite element method (FEM) design procedures.

Key words: vibration, ultrasonic assisted machining, ultrasonic horn, finite element method, modal properties

1. INTRODUCTION

Ultrasonic vibrations have been harnessed with considerable benefits for a variety of industrial applications. These applications include the automotive, food preparation, medical, textile and material joining and mainly applications in manufacturing industries.

The remarkable advantages are obtained by application of ultrasonic phenomenon in field of the machining of materials. The using of the ultrasonic vibration energy provides two different approaches to machining process. The first approach is based on application of ultrasonic transducer which is utilised indirectly to propel abrasive particles suspended in slurry at the work surface causing slow erosion. This principle of the machining is called as „ultrasonic machining (USM)“. The second approach is based on the transfer of the ultrasonic vibrations directly on the cutting tool, respectively directly to a cutting process. The second approach is known under term of „ultrasonic assisted machining (UAM)“. The advantages of this possibility, which is referred to subsequently as ultrasonic cutting/machining, are not obvious, because normally machine tool vibration has to be vigorously suppressed in the most cases [1]. The principle of UAM can be applied to different machining technologies, like a turning, grinding, boring, milling and others. The repetitive high-frequency vibro-impact mode brings some unique properties and improvements into metal cutting process [1], [2], [7], where the interaction between workpiece and the cutting tool is transformed into a micro-vibro-impact process.

In the ultrasonic machining systems, the electromechanical transducer acts as the source of mechanical oscillations, transforming the electrical power received from the generator into mechanical vibrations. The electromechanical transducers are based on the principle utilizing magnetostriction or piezoelectric effects. The electromechanical ultrasonic transducers generate the vibration with resonant frequency \( f_{res} \approx 20 \text{ kHz} \) and more. However, the amplitude of the resulting ultrasonic vibrations is inadequate for realization of the cutting process. To
overcome this problem, a wave-guide focusing device known as a horn (also known as concentrator or tool holder) is fitted onto the end of the transducer. The horn transfers the longitudinal ultrasonic waves from the transducer end to the toe end with attached the cutting tool. The cutting performance of ultrasonic machining equipment primarily depends on the well-taken design of the sonotrode [5]. It amplifies the input amplitude of vibrations so that at the output end the amplitude is sufficiently large to required machining process.

Positive results [3] in machining processes of hardly machinable materials (hard, soft, brittle and ductile materials) are obtained by using ultrasonic assisted machining. These positive results could not be achieved using conventional machining processes. The benefits of ultrasound applications in machining are related to a reduction in cutting forces, improved machined surface quality, reduction of tool wear and the resulting increasing his life cycle, etc.

In the present paper, a finite element method (FEM) design procedure was used to the study and to determine of modal properties of considered ultrasonic horns. The main aim of this paper is to present generally valid results leading to the geometrical design of sonotrode with effective dynamical properties.

2. ULTRASONIC HORN DESIGN

The ultrasonic horn is used as an element that serves to transfer of the vibration energy from the transducer towards to the tool interacting with workpiece. The principal function of the horn is to amplify the amplitude of ultrasonic vibration of the tool to the level required to the effective machining. It does so by being in resonance with the transducer. The horn design and its manufacture of require special attention. Incorrectly designed horn will impair machining performance and can lead to the destruction of the vibration system and cause considerable damage to the generator. Generally, the horns are made of metals that have high fatigue strengths and low acoustic losses. To the most often used metals to the horn manufacturing are monel, titanium, stainless steel, heat treated steel and aluminium.

The most important aspect of horn design is a horn resonant frequency and determination of the correct horn resonant wavelength, which should be usually integer multiple of the half wavelength of the system. The resonant frequency can be determined analytically (simple geometrical shape) or numerically (complicated geometrical shape). The required performance of ultrasonic horn is assessed by an amplification factor

$$\vartheta = \frac{A_2}{A_1}, \quad (1)$$

where $A_1$, resp. $A_2$ - amplitude of input end, resp. output end of horn. The fundamental horn design requirement is $\vartheta > 1$.

2.1 Mathematical model of vibrating horn

We suppose that a horn is made of isotropic material (Young’s modulus - $E$, density - $\rho$, Poisson’s ratio - $\nu$).

The governing equation of longitudinally free vibrating ultrasonic horn with variable circular cross-section can be expressed in following form

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c_p^2 \frac{\partial}{S(x) \partial x} \left[ S(x) \frac{\partial u(x,t)}{\partial x} \right] = 0, \quad (2)$$

where $x$ - coordinate, $u(x, t)$ - longitudinal displacement of cross-section, $c_p = \sqrt{E/\rho}$ - velocity of the longitudinal elastic waves propagation in horn material, $S(x) = S_0 f(x)$ - circular cross-section at coordinate $x$, $f(x)$ - function defining the cross-section change in longitudinal $x$ direction, $S_0$ - circular cross-section at coordinate $x = 0$.

Generally, the solution of equation of motion (2) can be supposed in the form

$$u(x, t) = U(x)e^{\text{ipo} t}. \quad (3)$$
The following non-dimensional quantities are introduced:
- dimensionless coordinate
  \[ \xi = \frac{x}{l_0} ; \quad \xi \in \langle 0; 1 \rangle, \]  \( \text{(4)} \)
- dimensionless longitudinal displacement
  \[ \overline{U}(\xi) = \frac{U(\xi)}{l_0}. \]  \( \text{(5)} \)

After substituting (3)-(5), the equation (2) as the following form
\[ \frac{1}{f(\xi)} \frac{d}{d\xi} \left[ f(\xi) \frac{dU(\xi)}{d\xi} \right] + \beta^2 \overline{U}(\xi) = 0. \]  \( \text{(6)} \)

The frequency parameter \( \beta \) is defined by
\[ \beta = \frac{\omega_0}{c_\rho l_0}, \]  \( \text{(7)} \)
where \( \omega_0 \) - natural angular frequency, \( l_0 \) - length of horn.

For the cylindrical shape of ultrasonic horn \( f(\xi) = \text{const.} \), the equation
\[ \frac{d^2 \overline{U}(\xi)}{d\xi^2} + \beta^2 \overline{U}(\xi) = 0, \]  \( \text{(8)} \)
has the solution in the form
\[ \overline{U}(\xi) = A \cos(\beta \xi) + B \sin(\beta \xi), \]  \( \text{(9)} \)

The integration constants \( A, B \) can be determined by boundary conditions \[^\text{[7]}\].

The boundary conditions for free vibration of horn are supposed (free edge on both sides of the horn) \[^\text{[7]}\] in the form
\[ \left. \frac{d\overline{U}(\xi)}{d\xi} \right|_{\xi=0} = 0, \quad \left. \frac{d\overline{U}(\xi)}{d\xi} \right|_{\xi=1} = 0. \]  \( \text{(10)} \)

Introducing (9) into boundary conditions given by (10), the following modal parameters of horn vibration are determined:
- natural frequency of the \( k^{th} \) mode shape
  \[ f_{0k} = \frac{k}{2l_0} \sqrt{\frac{E}{\rho}}, \quad k = 1,2,\ldots, \]  \( \text{(11)} \)
- wave length of the \( k^{th} \) mode shape
  \[ \lambda_k = \frac{2\pi}{\beta_k} = \frac{2}{k}, \quad k = 1,2,\ldots. \]  \( \text{(12)} \)

To the horn shape design only two cases are used. i.e. for \( k = 1 \) so-called “half wave” shape and \( k = 2 \) “wave” shape (see Fig. 1).

![Modal shapes of horn vibration](image)

\( \lambda_1 = 2, \quad f_{01} = \frac{1}{2l_0} \sqrt{\frac{E}{\rho}}, \quad \vartheta_1 = 1 \)
\( \lambda/2 \) - „half wave” shape

\( \lambda_2 = 1, \quad f_{02} = \frac{1}{l_0} \sqrt{\frac{E}{\rho}}, \quad \vartheta_2 = 1 \)
\( \lambda \) - „wave” shape

Fig. 1. Modal shapes of horn vibration

The analytical calculation of the resonant length and resonant frequency for the cylindrical shape of ultrasonic horn is simply problem. To the determination of modal properties for more complicated geometrical horn shapes, the numerical simulation \[^\text{[4]}\] is better to use.

2.2 FE modelling of horn vibration

The effects of variations in design variables on the modal properties (modal shapes, natural frequencies) in typical geometries of horns used for UAM is analysed by finite element method. The FEM modelling was done using the software package ANSYS.
The basic dynamic FEM equation of motion for free vibration of horn suitable to horn modal properties can be generally expressed as

$$\mathbf{M} \ddot{\mathbf{u}} + \mathbf{K} \mathbf{u} = 0 ,$$  \hfill (13)

where \( \mathbf{M} \), resp. \( \mathbf{K} \) - mass matrix, resp. stiffness matrix, \( \ddot{\mathbf{u}} \), resp. \( \mathbf{u} \) - vector of node acceleration, resp. node displacement.

The modal properties of horn are determined by the solution of eigenvalue problem

$$(\mathbf{K} - \omega_i^2 \mathbf{M}) \phi_i = 0 ,$$  \hfill (14)

where \( \phi_i \) - \( i \)th mode shape, \( \omega_i \) - natural angular frequency of \( i \)th mode shape. 

The horns can be manufactured in various shapes and dimensions. They are primarily of circular cross-section and the most common shapes are cylindrical, tapered and exponential (see. Table 1).

<table>
<thead>
<tr>
<th>Cylindrical</th>
<th>Tapered</th>
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<tbody>
<tr>
<td>slenderness ratio</td>
<td>( \delta = 2r_0/l_0 )</td>
</tr>
<tr>
<td>shape parameter</td>
<td>( \alpha \in (0\degree;15\degree) )</td>
</tr>
<tr>
<td>shape function</td>
<td>( f(\xi) = \frac{1}{1 - 1 - (1 - 2\delta^{-1}tg\alpha)\xi^2} )</td>
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Table 1. Geometrical parameters of horn shapes

In the Table 1, the parameter \( r_0 \) is radius of input end of horn and \( l_0 \) is length of horn, \( \alpha \) is incline angle of tapered horn shape and \( a \) is exponential base function.

The finite element models of considered shapes of ultrasonic horns, which are created by ANSYS code are shown in Table 1. The type of finite element is SOLID45.

3. ANALYSIS AND RESULTS

The numerical simulation to determine the modal properties of considered ultrasonic horn shapes are performed. The shapes used to the horn model creation and horn parameters are shown in Table 1. The steel as a horn material is used to numerical simulation (\( E = 210 \) GPa, \( \rho = 7800 \) kgm\(^{-3}\), \( \nu = 0.3 \)).

In the following, the non-dimensional natural frequencies for different geometrical shapes of horn are defined as

$$\Theta_i = \frac{f_i}{f_{0i,cyl}} ,$$  \hfill (15)

where \( f_i \) - \( i \)th natural frequency of analysed horn, \( f_{0i,cyl} \) - \( i \)th natural frequency of cylindrical horn. Both frequencies are considered for the same slenderness ratio. Then the value of resonant frequency of corresponding geometrical horn shape is determined using following equation

$$f_i = \Theta_i f_{0i,cyl} .$$  \hfill (16)

Cylindrical horn shape.

Fig. 2. Dependency of \( \Theta \) vs. slenderness ratio \( \delta \) for cylindrical horn shape.
Tapered horn shape.

Exponential horn shape.

Fig. 3. Dependency of $\theta_1$ (resp. $\theta_2$) vs. slenderness for different incline angles

Fig. 5. Dependency of $\theta_1$ (resp. $\theta_2$) vs. slenderness for different exponential bases

Fig. 4. Dependency of $\theta_1$ (resp. $\theta_2$) vs. slenderness for different incline angles

Fig. 6. Dependency of $\theta_1$ (resp. $\theta_2$) vs. slenderness for different exponential bases
4. CONCLUSIONS

A general procedure for design of high-frequency ultrasonic horn for ultrasonic manufacturing technologies is presented in this paper. By use of wave theories for longitudinal vibration of rod and FEM analysis, the effect of horn geometrical shape for ultrasonic manufacturing technologies is analysed. The different geometrical horn shapes (cylindrical, tapered and exponential) were considered. The effects of horn shape parameters such as a slenderness ratio \( \delta \), incline angle \( \alpha \), base of exponential function \( a \) on natural frequencies had been analysed. The results presented in this paper give the possibilities to design of ultrasonic horn with required dynamical properties. Using Eq. (16), the required value of resonant frequency is determined. Note that the fundamental requirements required for the application of considered ultrasonic horns - resonance frequency have to be more than 20 kHz and amplification factor is \( \vartheta > 1 \).

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6. REFERENCES


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