Abstract: Composing of multi-pole models and simulation of dynamic responses of a technical chain system is considered in the paper. In Part 1 of the paper difficulties arising in using existing simulation tools have been discussed. A methodology has been proposed that seems to be free of most of these disadvantages. Modeling of electro-hydraulic servovalve has been considered as an example of chain system. An intelligent simulation environment CoCoViLa supporting declarative programming in a high-level language and automatic program synthesis is used as a tool for modeling and simulation. In Part 2 multi-pole mathematical models of functional elements are described. Computing transient responses of the servovalve are considered.

Key words: multi-pole model design, electro-hydraulic servovalve, intelligent programming environment, simulation.

1. INTRODUCTION

Electro-hydraulic servovalve has control function in electro-hydraulic servo-systems that are used in various applications, including military and commercial aircraft flight controls, satellite positioning controls, controls for steering tactical and strategic missiles. They are also used in industrial applications, including injection molding machines for the plastics markets, metal forming, robots and manipulators, synchronized drives, power generating turbines, simulators used to train pilots etc. Computer modeling and simulation is the first step in the design of such systems.

The electro-hydraulic servo-system is controlled by an electro-hydraulic servovalve [1-7]. Sensors are used for feedback, regulator is used for creating and modifying the control signal.

2. MULTI-POLE MATHEMATICAL MODEL OF AN ELECTRO-HYDRAULIC SERVOMOTOR

The multi-pole model of an electro-hydraulic servovalve is decomposed into three components – torque motor with flapper TM, nozzle-and-flapper valve NF and spool in sleeve with elastic feedback to flapper SP (Fig.1).

Fig. 1. Multi-pole model of an electro-hydraulic servovalve

2.1 Multi-Pole Mathematical Model of a Torque Motor TM

The input variables of the torque motor TM (see Fig. 1) are the input voltage $U$, hydrodynamic force moment of the fluid jets to the flapper $Thd$ and force $Fz$, acting to the sliding spool. The output variables of the torque motor are anchor rotating angle $th1$ from central position,
linear displacement of anchor \(x1\) and current \(I\).

Mathematical dependences of torque motor TM for statics:

\[
I = \frac{U}{R};
\]

\[
J_{tu} = \frac{P_i^*(\text{dtu})^4/64*(1 - (\frac{ds}{\text{dtu}})^4)}{2*E*J_{tu}/\text{ltu}};
\]

\[
ctu = \frac{2*E*J_{tu}/\text{ltu}}{\text{ltu}^2*}
\]

\[
(ktm*I-Fz*ltu-Thd)/2;
\]

\[
x1 = \frac{1}{3*E*J_{tu}*(\text{ltu}^2*)};
\]

\[
(ktm*I-Fz*ltu-Thd)).
\]

Additional equation for dynamics:

\[
U_{in} = U - koe*om.
\]

Differences for computing of Runge-Kutta coefficients:

\[
dI = \frac{(U_{in} - R*I)/L*\delta}{\delta};
\]

\[
dom = \frac{(\frac{(ktm*I-I_{tu}*Fz-Thd)/2 - bvr*om - (ctu - ctm)*th}{\delta})/(Itm + Imf)}{\delta};
\]

\[
dth = om*\delta.
\]

2.2 Multi-pole mathematical model of a nozzle and flapper valve NF

The input variables of a nozzle-and-flapper valve NF (see Fig.1) are torque motor anchor rotating angle \(th_1\), anchor feedback rotating angle \(th_2\), velocity of the sliding spool \(v\), feeding pressure \(p_1\) and output pressure \(p_{10}\). The output variables are the pressure difference of the sliding spool \(dp\), hydrodynamic force moment of the fluid jets to the flapper \(Thd\), displacement of the flapper \(h\), volumetric flow rate in inlet \(qV1\) and in outlet \(qV8\).

Detailed multi-pole block scheme of a nozzle-and-flapper valve NF is shown in Fig. 2.

Mathematical dependences of nozzle-and-flapper valve NF are as follows.

Equations for calculation of the areas:

- for spool \(A = \frac{P_i*d_{sp}^2}{4}\\
\)
- for nozzles \(A_{noz} = \frac{P_i*d_{noz}^2}{4}\\
\)
- for resistors \(R1...R8\)

\[
Ar1...Ar8 = \frac{\rho_{1...8}}{2/(\mu_{1}*(Ar1...Ar8))}^2.
\]

\[
RL1...RL8 = AL*(lr1...lr8)*(nue1...nue8)*\frac{\rho_{1...8}}{P/(Ar1...Ar8)}^2/8.
\]

The physical properties of fluid (density \(\rho\) and viscosity \(\nu e\)) are determined at constant temperature in dependence on resistor input pressure at each calculation step.

Fig. 2. Multi-pole block scheme of a nozzle-and-flapper valve NF, where \(N1, N2\) - nozzles, \(R1...R8\) - hydraulic resistors, \(IE1...IE4\) - interface elements (tee couplings), \(dp\) - difference of pressures on the ends of sliding spool, \(F_{hd}\) - sum of hydrodynamic forces of the fluid jets to the flapper, \(th\) – difference of inlet and feedback angular displacements of the TM anchor

Mathematical dependences of displacement of the flapper \(h\), hydraulic conductivities \(Gn1, Gn2\) and linear resistances \(RLn1, RLn2\) of nozzle-and-flapper valve:
\[ h = (l_1 - l_{tu})*(th_1 - th_2); \]
\[ Gn_1 = (\text{munoz}*[\text{Pi}*d\text{noz}*(\text{hsymm} - h))^2*\text{rho}/9; \]
\[ RL_{n1} = AL*[ln_{nue9}*[\text{nue9}*[\text{rho9}]; \]
\[ Gn_2 = (\text{munoz}*[\text{Pi}*d\text{noz}*[\text{hsymm} + h)]^2/2/\text{rho10}; \]
\[ RL_{n2} = AL*[ln_{nue10}*[\text{nue10}*[\text{rho10}]; \]

For calculating volumetric flow rates and pressures in the servovalve it is necessary to determine the approximate initial values of volumetric flow rates through nozzles. This can be done using non-linear oriented graph of the nozzle-and-flapper. To perform calculations more efficiently it is appropriate to transform the oriented graph into linear signal flow graph operating with volumetric flow rate square values (variables identified by prefix “w”). Iterations are used for calculating approximate initial values of volumetric flow rates through nozzles.

The output values are expressed as:

\[ h = (l_1 - l_{tu})*(th_1 - th_2); \]
\[ dp = p_5 - p_6; \]
\[ F_{hd} = ((p_7 - p_8)*\text{Anoz} + (\text{wqVn1} - \text{wqVn2})*\text{rho}/\text{Anoz}); \]
\[ Thd = l_{tu} * F_{hd}; \]
\[ qV_1 = (\text{wqV1})^2; \]
\[ qV_8 = (\text{wqV8})^2. \]

### 2.3 Multi-pole mathematical model of a spool in sleeve with elastic feedback SP

The input values of a multi-pole model SP (see Fig. 1) are \( dp, F_{hd}, th_1, x_1, F_{13} \) and \( F_{24} \). The output values are \( th_2, v \) and \( z \).

Mathematical dependences of statics of sliding spool with elastic feedback:

\[ A = \text{Pi}*[dz]^2/4; \]
\[ F_z = A*dp; \]
\[ T_z = l_{tu}*F_z; \]
\[ Thd = l_{tu}*F_{hd}. \]

It is necessary to solve the integral to determine bending moment of inertia of the conic feedback spring:

\[ I = \int_0^1 \left( (d_2 - 2*x*tg\alpha)^4 + dx, \right.), \]
where \( 1 = l_{2} - l_{3}; \)
\[ \text{tg}\alpha = (d_2 - d_1)/(2*l). \]

The integral can be expressed as:

\[ I = l/(5*(d_2 - d_1)*((d_2^5 - d_1^5)), \]
where \( (d_2^5 - d_1^5) = (d_2 - d_1)* \]
\( (d_1^4 + d_1^3*d_2 + d_1^2*d_2^2 + d_1*d_2^3 + d_2^4). \]

The bending moment of inertia of the conic feedback spring:

\[ J_{f} = \text{Pi}*[d_1^4 + d_1^3*d_2 + d_1^2*d_2^2 + d_1*d_2^3 + d_2^4]/(5*64). \]

The bending moment of inertia of the flexure tube:

\[ J_{tu} = \text{Pi}*[d_{tu}^4]/64*(1 - (ds/d_{tu})^4). \]

The values of \( x_2, x \) and \( th_2 \) are:

\[ x_2 = 1/(3*E*J_{tu})*(l_{tu}^2)*(T_z + Thd); \]
\[ x = x_1 - x_2; \]
\[ th_2 = l_{tu}/(2*E*J_{tu})*(T_z + Thd). \]

The sliding spool displacement:

\[ z_c = 1/(3*E*J_{f})*(l_{2} - l_{3})^3*F_z - (l_1 + l_2)*(th_1 - th_2) + x. \]

The bending rigidity of feedback spring:

\[ c_z = (3*E*J_{f})/(l_{2} - l_{3})^3. \]

Differences for computing Runge-Kutta coefficients:

\[ dv = (F_z - F_{fr} - hsp*v - cz*(z + l^*(th_1-th_2c) - x)/\text{msp})*\delta; \]
\[ dz = v*\delta. \]

### 3. SIMULATING OF DYNAMICS

The simulation task description of an electro-hydraulic servovalve dynamics in CoCoViLa environment [8] is shown in Fig. 3.

**Multi-pole models of functional elements:**
- **TM** – torque motor,
- **NFD** – nozzle-and-flapper valve,
- **SP** – sliding spool in sleeve with elastic feedback from spool to flapper.
Fig. 3. Simulation task description of an electro-hydraulic servovalve dynamics

**Inputs:** dynamic Source – input disturbances (voltage \(U\), feeding pressure \(p_1\)), dyn_stat Source (\(p_1\) mean value, initial volumetric flow through nozzle \(iqVn1\)); constant Source – constant values (outlet pressure \(p_{10}\), hydrodynamic forces of fluid jets \(F_{13}\) and \(F_{24}\) acting to sliding spool).

**Outputs:** current \(I\), displacement \(h\) of the flapper, displacement \(z\) of the sliding spool.

**Iterated variables:** \(TM.th1\), \(TM.x1\), \(SP.th2\), \(SP.v\).

**Simulated graphs:** input voltage \(U\), displacement \(h\) of the flapper, displacement \(z\) of the sliding spool.

**Simulation manager:** dynamic Process 3D.

**Parameters for TM:** \(ctm = 10\), \(ds = 0.003\), \(dtu = 0.0038\), \(E = 2.1E11\), \(Imax = 5e-2\), \(ktm = 3, 11 = 0.0105\), \(I_2 = 0.020\), \(I_3 = 0.004\), \(ltu = 0.0075\), \(zmax = 6.8E-4\), \(hsp = 15\), \(msp \leq 1E-2\).

In the examples dynamic behavior of the servovalve is simulated depending on step and jump disturbances of input voltage.

A special simulation engine has been used for performing simulations and calculating dependences on two different arguments (time and disturbance step in current example). In the examples several dependences can be calculated and presented simultaneously.

Simulation results are shown in Fig. 4...7. In Fig. 4 the results of the simulation are shown when two different input step disturbances (voltage \(U = 2\) and \(8\) V) during \(tstep = 0.01\) s (graphs 1) are applied and \(p_1 = 2.1E7\) Pa, \(p_{10} = 1.5E6\) Pa, \(F_{13}, F_{24} = 0\) N. Displacements \(h\) of the flapper (graphs 2) in interval of \(0...tstep\) follow the input disturbances. In the interval from \(tstep\) to \((0.02, 0.04\) s) the flapper takes a new position (\(8E-8, 28E-8\) m) due to feedback. Displacements \(z\) of the sliding spool (graphs 3) increase from \(0\) to \((1.5E-4, 5.8E-4\) m) during \((0.022, 0.039\) s). The results are in accordance with catalog characteristics [9].

Fig. 4. Simulated graphs of the electro-hydraulic servovalve for step responses (input voltage step \(U = 2\) and \(8\) V)

In Fig. 5 the results of the simulation are shown when the input step disturbance (voltage \(U = 8\) V) during \(tstep = 0.01\) s (graphs 1) is applied and \(p_{10} = 1.5E6\) Pa, \(F_{13}, F_{24} = 0\) N. The graphs are calculated for two different values of feeding pressure \(p_1 = 7E6\) and \(21E6\) Pa.

Displacements \(h\) of the flapper (graphs 2) in interval of \(0...tstep\) follow the input disturbances. In the interval from \(tstep\) to \((0.045, 0.040\) s) the flapper takes a new
position (59E-8, 28E-8 m) due to feedback. Displacements $z$ of the sliding spool (graphs 3) increase from 0 to (3.5E-4, 5.8E-4 m) during (0.045, 0.040 s).

In Fig. 6 the results of the simulation are shown when two different input jump disturbances (voltage $U = 2$ and 8 V, tstep1 = tstep2 = 0.01 s, tjum = 0.02 s) (graphs 1) are applied and $p1 = 2.1E7$ Pa, $p10 = 1.5E6$ Pa, $F13 = F24 = 0$ N. The graphs are calculated for two different values of feeding pressure $p1 = 7E6$ and 21E6 Pa.

A result to be pointed out is that in case of a particular input jump disturbance the displacement of flapper (graphs 2) is less when the feeding pressure is higher. Displacement $z$ of the sliding spool (graphs 3) is greater when the feeding pressure is higher.

**CONCLUSIONS**

In the Part 1 of the paper principles of multi-pole modeling of technical chain systems have been described. Modeling and simulation of an electro-hydraulic servovalve has been considered as an example. An intelligent simulation environment CoCoViLa supporting declarative programming in a high-level language and automatic program synthesis is used as a tool for modeling and simulation.

A special technique has been proposed and used that allows to avoid solving large equation systems during simulations. Therefore, multi-pole models of large systems do not need considerable simplification.

In the current paper multi-pole mathematical models are considered in
detail. Results of dynamic simulations have been presented and discussed. The proposed modeling and simulation procedure is an efficient and powerful tool in design of technical chain systems.

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NOMENCLATURE

Nomenclature for torque motor TM with flapper

**Input variables (input poles):**
- $F_z$: Force, acting to the sliding spool, N
- $Thd$: Torque evoking through hydrodynamic forces of the fluid jets to the flapper, acting onto anchor of the TM, Nm
- $U$: Input voltage of the TM, V

**Output variables (output poles):**
- $I$: Current to the TM, A
- $th1$: TM anchor rotating angle, rad
- $x1$: Horizontal bending displacement of the flexure tube, m

**Inner variables:**
- $\omega_m$: TM anchor rotating angle velocity, rad/s
- $U_{in}$: Input voltage to the TM in dynamics, V

**Differences for computing of Runge-Kutta coefficients:**
- $dI$: Difference of current to the TM, A
- $d\omega_m$: Difference of angular velocity of the TM anchor, rad/s
- $dth1$: Difference of TM anchor rotating angle, rad

**Parameters:**
- $b_{vr}$: Viscous angular resistance coefficient of the anchor of TM, Nms/rad
- $ctm$: Angular stiffness of the “magnet spring” of the TM, Nm/rad
- $ds$: Flexure tube inner diameter, m
- $dtu$: Flexure tube outer diameter, m
- $E$: Modulus of elasticity, N/m²
- $l_{max}$: Max. current in the TM, corresponding to max. displacement of the spool (z_{max}), A
- $l_{tm}$: Moment of inertia of the TM anchor, Nms²
- $l_{fl}$: Moment of inertia of the flapper, reduced to the axis of the TM, Nms²
- $k_{oe}$: Coeff. of the opposite electromotor force, Vs/rad
- $ktm$: Coeff. of the moment characteristic of the TM, Nm/A
- $ltu$: Flexure tube length, m
- $l_l$: Distance between the TM anchor and the nozzle axis, m
- $l_r$: Distance between the nozzle axis and feedback spring end on the spool, m
- $L$: Inductivity of the circle, H
- $R$: Actual resistance of the circle, Ω

**Parameters to be calculated:**
- $ctu$: Angular bending stiffness of the flexure tube, Nm/rad
- $J_{tu}$: Bending moment of inertia of flexure tube, m⁴

Nomenclature for nozzle-and-flapper valve NF

**Input variables (input poles):**
- $p_1$: Feeding pressure of servovalve, Pa
- $p_{10}$: Outlet pressure of the servovalve, Pa
- $th1$: Inlet angular displacement of the TM anchor, rad
- $th2$: Feedback angular displacement of the TM anchor, rad
- $v$: Velocity of the sliding spool, m/s

**Output variables (output poles):**
- $dp$: Difference of pressures on the ends of sliding spool, Pa
- $Fhd$: Sum of hydrodynamic forces of the fluid jets to the flapper, N
- $h$: Shift of flapper from symmetric position, m
- $qV1$: Feeding volumetric flow rate, m³/s
- $qV8$: Outlet volumetric flow rate, m³/s
- $Thd$: Torque evoking through hydrodynamic force of the fluid jets to the flapper, acting onto anchor of the TM, Nm

**Inner variables:**
- $F_z$: Force, acting to sliding spool, N
- $p_{2...9}$: Pressures, Pa
- $qV_{2...7}$: Volumetric flow rates through resistors, m³/s
- $qV_{n1}, qV_{n2}$: Volumetric flow rates through nozzles, m³/s
- $wqV_{1...7}$: Volumetric flow rates through resistances in square, m⁶/s²
- $wqV_{n1}, wqV_{n2}$: Volumetric flow rates through nozzles in square, m⁶/s²

**Parameters:**
- $AL$: Hydraulic friction coefficient of laminar flow
- $dnoz$: Diameter of nozzles, m
- $dsp$: Diameter of sliding spool, m
- $dr_{1...8}$: Diameters of hydraulic resistors, m
- $hs_{symm}$: Distance between flapper and nozzle in symmetric position of flapper, m
- $l_{1}$: Distance between the TM anchor and the nozzle axis, m
- $lr_{1...8}$: Lengths of fluid jets through resistors
- $l_{r1}, l_{r2}$: Lengths of fluid jets through nozzles

N1,N2 for computing of linear resistances, m
ltu  Bending length of the flexure tube, m  
\( \mu_1 \)  Discharge coefficients of resistors R1, R4…R8  
\( \mu_2 \)  Discharge coefficients of resistors R2, R3  
\( \mu_{\text{noz}} \)  Discharge coefficient of nozzles N1, N2  

**Parameters to be calculated:**  
\( A \)   Active areas of spool at the ends, m²  
\( A_{\text{noz}} \)  Passage area of the nozzles, m²  
\( A_{r1} \ldots A_{r8} \)  Passage areas of resistors R1…R8, m²  
\( \eta_{\text{n1}}, \eta_{\text{n2}} \)  Hydraulic conductivities of nozzle-and-flapper valve, m³/kg  

Nomenclature for spool in sleeve SP with elastic feedback  

**Input variables (input poles):**  
\( dp \)  Difference of pressures on the ends of spool, Pa  
\( F_{13} \)  Hydrodynamic force of fluid jets through slot 1 and 3, N  
\( F_{24} \)  Hydrodynamic force of fluid jets through slot 2 and 4, N  
\( F_{hd} \)  Hydrodynamic force of the fluid jets to the flapper, N  
\( \theta_{\text{i}} \)  Inlet angular displacement of the TM anchor, rad  
\( x_1 \)  Linear displacement of the flexure tube, m  

**Output variables (output poles):**  
\( F_{z} \)  Force, acting to the sliding spool, N  
\( \theta_{2} \)  TM anchor rotating angle evoking through feedback, rad  
\( v \)  Sliding spool velocity, m/s  
\( z \)  Sliding spool displacement, m  

**Inner variables:**  
\( T_{hd} \)  Torque evoking through hydrodynamic force of the fluid jets to the flapper, Nm  
\( T_{z} \)  Torque evoking through force, acting to sliding spool, N  
\( x_2 \)  Horizontal bending displacement of the flexure tube, m  

**Differences for computing of Runge-Kutta coefficients:**  
\( dv \)  Difference of sliding spool velocity, m/s  
\( dz \)  Difference of sliding spool displacement, m  

**Parameters:**  
\( d_1 \)  Feedback conic spring litter diameter, m  
\( d_2 \)  Feedback conic spring major diameter, m  
\( ds \)  Flexure tube inner diameter, m  
\( d_{tu} \)  Flexure tube outer diameter, m  
\( dz \)  Sliding spool diameter, m  
\( E \)  Modulus of elasticity, N/m²  
\( F_{fr} \)  Friction force of sliding spool, N  
\( h_{\text{max}} \)  Maximum displacement of the flapper from symmetric position, m  
\( h_{sp} \)  Damping coefficient of the sliding spool, Ns/m  
\( l_{tu} \)  Flexure tube length, m  
\( l_{1} \)  Distance between the TM anchor and the nozzle axis, m  
\( l_{2} \)  Distance between the nozzle axis and feedback force from spool, m  
\( l_{3} \)  Distance between the nozzle axis and end of rigid part of flapper, m  
\( l \)  Distance between the TM anchor and feedback force from spool, m  
\( m_{sp} \)  Mass of sliding spool, kg  
\( z_{\text{max}} \)  Maximum position of sliding spool from initial position, m  

**Parameters to be calculated:**  
\( A \)  Active areas of spool at the ends, m²  
\( c_{tu} \)  Rotating rigidity of the flexure tube, Nm/rad  
\( c_{z} \)  Feedback spring bending rigidity, N/m  
\( J_f \)  Bending moment of inertia of the feedback spring, m⁴  
\( J_{tu} \)  Bending moment of inertia of the flexure tube, m⁴  

**Calculation parameters:**  
\( \delta \)  Time step, s  
\( \tau \)  Inverse value of the time step, (1/s)  

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