Abstract: In this paper an alternative approach to solving the inverse problem of kinematics is observed. In comparative analysis ways for determining link’s rotatory angles using classical and alternative approaches will be observed. It will allow to determine advantages of using the alternative approach proposed in this article. On the basis of the obtained results we can define ways for optimizing the algorithm for solving the inverse problem of kinematics in future.

Key words: manipulator, algorithm, trajectory, the inverse problem

1. INTRODUCTION

Commonly during the design of a manipulator only required characteristics of technological process and manipulator’s productivity are known. Thus, when modifying a manipulator and choosing the number of its links, their optimal length and characteristics of electrical drives, appears the necessity to solve the inverse problem of kinematics.

Nowadays the inverse problem of kinematics is used in the variety of different areas of life. Among them are war industry, technological industry, cinematograph etc. Because of the theoretical and practical difficulties of the inverse kinematics there is a big area for explorations and making optimizations [2].

In the first approach, the solution of the inverse problem is based on a method of approximations [3]. We have randomly determined these approximations from the equation of motion of the first link’s movement at time (t=0). According to the determined equation of motion of the grip, the kinematical characteristics of all links at subsequent times are found.

The second approach is based on a geometrical method. The main idea is to identify the extreme possible positions of links from the determined equation of motion of grip. These extreme positions of links are obtained by replacement of the initial system with a simplified model. Derived algorithm allows to provide sufficient accuracy for a motion of the grip by determined trajectory. The developed algorithm allows to consider manipulator with different number of links and to simulate continuity, smoothness and the minimal time of link’s (grip’s) motion by determined trajectory. The 3D mathematical model generated in MATLAB is proposed for simulation of manipulator’s movement as a graphical solution of inverse problem of kinematics.

2. INVERSE KINEMATICS

The main problem of the inverse kinematics is to find connected parameters \( \theta_1 = [\theta, \theta_2, \theta_3, ... \theta_n] \) of manipulator, providing set orientations of grip using known positional matrix \( T^0_6 \) and orientation of grip of 6-link manipulator (Fig.1) [4].

So, we have:

\[
R^0_6 = B^{-1}R^\text{abs}_{\text{inst}}H^{-1} \begin{bmatrix} n_x & s_x & a_x & \rho_x \\ n_y & s_y & a_y & \rho_y \\ n_z & s_z & a_z & \rho_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
\] (1)
where: \( H \)-matrix of position in coordinates system of grip, \( B \)-matrix of position in absolute coordinate system and \( R_{\text{instr}}^{\text{abs}} \) - matrix of position of an instrument in absolute coordinate system, \( n \) - vector of normal, \( s \) - vector of orientation, \( a \) - vector of axis of rotation.

### 2.1 Classical approach to inverse kinematics

Let we see link frame assignment of robot “Fanuc S-500” [5].

From Fig.1 and initial conditions of inverse kinematics we know vector \( P \) (the grip’s position) and matrix of grip’s position and orientation \( R_{0}^{0} = [n \; s \; a] \), we can find the wrist position [1]

\[
(P_{w}) : \quad P_{w} = P - d_{6} \cdot a . \tag{2}
\]

It is now possible to find the inverse kinematics for \( \theta_{1} \), \( \theta_{2} \) and \( \theta_{3} \). Considering Fig. 2 it is easy to see that

\[
\theta_{1} = \arctan\left(\frac{P_{wy}}{P_{wx}}\right). \tag{3}
\]

Once \( \theta_{1} \) is known the problem reduces to solving a planar structure. Looking to Fig. 2 it is possible to successively write

\[
\begin{align*}
P_{wx_{1}} &= \sqrt{P_{wx}^{2} + P_{wy}^{2}} \tag{4} \\
P_{wz_{1}} &= P_{wz} - d_{1} \tag{5} \\
P_{wx_{1}'} &= P_{wx} - a_{1} \tag{6} \\
P_{wy_{1}'} &= P_{wy} \tag{7} \\
P_{wz_{1}'} &= P_{wz} \tag{8} \\
P_{wx_{1}'} &= a_{2}s_{2} + a_{x}s_{2y} \tag{9} \\
P_{wz_{1}'} &= a_{2}s_{2} + a_{x}s_{2y} \tag{10}
\end{align*}
\]

where coefficients \( c_{2} = \cos \theta_{2} \), \( s_{2} = \sin \theta_{2} \), \( c_{2y} = \cos(\theta_{2} + \theta_{y}) \), \( s_{2y} = \sin(\theta_{2} + \theta_{y}) \), \( a_{1}, a_{2}, a_{3}, d_{1}, d_{4}, d_{6} \) - distances from dimension sheet of robot, \( a_{x} \) - vector connecting points \{3\} and \{5\}, \( P_{wx}, P_{wy}, P_{wz} \) are dimensions in coordinate system \( Ox_{0}, y_{0}, z_{0} \), \( P_{wx_{1}}, P_{wy_{1}}, P_{wz_{1}} \) in coordinate system \( Ox_{1}, y_{1}, z_{1} \), \( P_{wx_{1}'}, P_{wy_{1}'}, P_{wz_{1}'} \) in coordinate system \( Ox_{1}', y_{1}', z_{1}' \) in Fig. 2.
Squaring and summing Eq. (9) and Eq. (10) results in

\[ p_{wx}^2 + p_{we}^2 = a_x^2 + a_y^2 + a_x a_y s_y. \]  

(11)

It gives

\[ s_y = \frac{p_{wx}^2 + p_{we}^2 - a_x^2 - a_y^2}{2a_xa_y}. \]  

(12)

Setting

\[ c_y = \pm \sqrt{1 - s_y^2} \]  

(13)

the solution for \( \theta_3 \) will be

\[ \theta_3 = \arctan(s_y, c_y) \]  

(14)

\[ \theta_3 = \theta_3 - \arctan \left( \frac{a_y}{d_y} \right). \]  

(15)

Now using \( \theta_3 \) in Eq. (9) and Eq. (10) results in a system with 2 equations with \( s_2 \) and \( c_2 \) unknowns

\[ p_{wx} = -a_x c_2 + a_y (c_y c_x - s_y s_y) \]  

(16)

\[ p_{we} = a_x s_2 + a_y (s_y c_x + s_x c_y). \]  

(17)

Solving for \( s_2 \) and \( c_2 \) gives

\[ s_2 = \frac{(a_y + a_x s_y) p_{wx} + a_x c_y p_{we}}{a_x^2 + a_y^2 + 2a_x a_y s_y}. \]  

(18)

\[ c_2 = \frac{(a_x + a_y s_y) p_{wx} + a_x c_y p_{we}}{a_x^2 + a_y^2 + 2a_x a_y s_y}. \]  

(19)

And the solution for \( \theta_2 \) will be

\[ \theta_2 = \arctan(s_2, c_2). \]  

(20)

To find the required joint angles \( \theta_4, \theta_5 \) and \( \theta_6 \), we simply take advantage of the special configuration of the last 3 joints, getting matrix of rotation

\[ R_6^3 = \left( R_3^0 \right)^{-1} \cdot R_6^0 = \left( R_3^0 \right)^T \cdot R_6^0, \]  

(21)

which gives

\[ R_6^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \]  

(22)

\[ r_{11} = -c_3 s_23 a_{11} - s_1 s_23 a_{21} + c_2 a_{31} \]

\[ r_{13} = -c_3 s_23 a_{13} - s_1 s_23 a_{23} + c_2 a_{33} \]

\[ r_{33} = s_1 a_{13} - c_1 a_{23} \]

\[ r_{21} = -c_3 c_23 a_{11} - s_1 c_23 a_{21} - s_3 a_{31} \]

\[ r_{31} = s_1 a_{11} - c_1 a_{21} \]

\[ r_{12} = -c_3 c_23 a_{12} - s_1 c_23 a_{22} + c_2 a_{32} \]

\[ r_{22} = -c_3 c_23 a_{12} - s_1 c_23 a_{22} + c_2 a_{32} \]

\[ r_{32} = s_1 a_{12} - c_1 a_{22}, \]

where \( a_{11}...a_{33} \) are elements of matrix \( R_6^0 \).

It is now possible to use the previous result for the ZYZ Euler angles to obtain the solutions for \( \theta_4, \theta_5 \) and \( \theta_6 \).

For \( \theta_2 \in [0, \pi] \) the solution is

\[ \theta_4 = \arctan(r_{33}, r_{13}) \]  

(23)

\[ \theta_5 = \arctan(\sqrt{r_{13}^2 + r_{33}^2}, -r_{23}) \]  

(24)

\[ \theta_6 = \arctan(-r_{22}, r_{21}). \]  

(25)

For \( \theta_2 \in [-\pi, 0] \) the solution is

\[ \theta_4 = \arctan(-r_{33}, -r_{13}) \]  

(26)

\[ \theta_5 = \arctan(-\sqrt{r_{13}^2 + r_{33}^2}, r_{23}) \]  

(27)

\[ \theta_6 = \arctan(r_{22}, -r_{21}). \]  

(28)

2.2 Alternative approach to inverse kinematics

For solution of the inverse kinematics the following functions were developed: «off», «rotate_time», «rotate_d», «time_angles». Algorithm of their use is shown in the block-scheme Fig. 6.
2.2.1 Function «of6»

Function «of6» is the fundamental function of algorithm for solution of inverse kinematics. Let us observe movement of manipulator using the system of links OABC in plane xz Fig. 3.

![Fig. 3. Input data](image)

We can divide all cycle of «of6» function into few steps:
1) Function «of6» gets input data with parameters of manipulators links; number of observing link; coordinates of the start point of observing link and the grip’s point of manipulator in this time.
2) Function determines the external possible positions of observing link and the maximal angle of its rotation \( b_3 \) by control of reaching the point of manipulator’s grip (point can be out of reach – too close or too far).
3) Function determines angles \( b_1, b_2, c_1 \) taking into consideration the maximal possible angle of rotation \( b_3 \).

If observing link is not the last Fig. 4 \( \rightarrow \) function repeats to all remaining links with changed initial coordinates to \( x'z' \) etc. As example angles can be found from Eq. (29)

\[
c_1 = b_1 + \frac{2}{3b_2}, \tag{29}
\]

where \( 2/3 \) is exemplifying numerical coefficient for gradation of angles \( c_1, b_1, b_2 \).

If observing link is the last Fig. 5 \( \rightarrow \) we can not use previous equation for angles got from the previous link and there is only one solution

\[
c_2 = b_1 - b_2. \tag{30}
\]

Thus, function «of6» determines angles of rotation of manipulator for initial plane \( xz \).

![Fig. 4. Solution for every link (except the last one)](image)

Finally, the obtained results in function «of6» are controlled by main program Fig. 6. After that the optimal variations of angles rotations are defined.

2.2.2 Function «rotate_time»

Function «rotate_time» finds the minimal time for rotation of every link in 2 required positions for some moment of time. Trajectory of manipulator consists of 3 mechanical characteristics of links motion: acceleration, inhibition, displacement with constant velocity.

Obviously, the minimal time of rotation of manipulator can be reached by having the highest possible velocity and acceleration of every link. Developing algorithm of function «rotate_time» permits to find the
weakest (slowest) links of manipulator. Analyzing that and changing mechanical characteristics of the weakest links, we obtain the optimized solutions.

2.2.3 Function «rotate_d»

Function «rotate_d» gets input data from function «rotate_time» and finds velocities and accelerations of links and selects the maximal time of movement for every link between two calculated positions. Moreover function «rotate_d» provides continuous movement of all the links of manipulator.

2.2.4 Function «time_angles»

It is the last basic functions of algorithm. It gives visualization for movement of manipulator and its link in every moment.

2.2.5 Simulation program for movement of manipulator

Simulation program for movement of manipulator is algorithm allows to determine velocities and accelerations of grip’s point in dependence on time moment by preliminarily selected trajectory. Currently there are 6 trajectories for manipulators movement demonstration. In Fig. 6 you can see full algorithm for alternative solution of inverse kinematics.

All required data for simulation is in massive data{k}(i,1:7), where 1-7 are parameters of link’s angles, velocities, accelerations etc.

In Fig. 7 and Fig. 8 there are few simulated positions of manipulator for some trajectories in 3D space created in MATLAB. The base of manipulator and its rotation around axis z is not depicted, but it was calculated together with other links.
3. ADVANTAGES OF ALTERNATIVE APPROACH

From description of classical and alternative approaches we can emit few important advantages of alternative approach:

- Possibility to optimize velocities and accelerations of manipulator’s links in dependence of parameters of links (length, weight, power of driving motors etc) to get the minimal possible time for every task
- Possibilities in more simple correction of rotatory angles to complete some complicated tasks (reach of point placed behind an mechanical barrier etc)
- Possibility to work out common algorithm of manipulator’s movement to solve both inverse and direct kinematics

4. CONCLUSION

Analyzing the main principles of alternative approach for inverse kinematics solution, we got some possible advantages to be compared to classical approach. It defines the main directions of development of proposed algorithm. To the next tasks for manipulator’s movement algorithm optimization we can relate calculation of additional trajectories and working out the most flexible mathematical solutions for calculation angles, velocities and accelerations for n-link manipulators. Well-modernized alternative approach can find its usage in different areas of life saving time of technological cycles, making economy in its usage of electric power, making solution for more and more complicated practical tasks.

5. REFERENCES


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