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MINIMISING BACK ORDERS IN A MULTI-ITEM PRODUCTION-INVENTORY SYSTEM

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Abstract: *This paper considers short-term production planning in a multi-item factory with capacity constraints. The factory produces items that are shipped to a warehouse. The warehouse has an uncertain daily demand for each of the item types. Some items have a nearly constant demand, but others are purchased occasionally. We try to minimise the number of back-ordered items, while keeping the stock size reasonable. To achieve this, we compare target inventory level- and production priority rule-based solutions. In the examples initial stock sizes range from the target level to a situation where there are no stocks. Simulations show that, in our problem, priority rules that take into account the whole demand distribution are better than average-based ones.*
Key words: production planning, make-to-stock (MTS), base stock system, priority, simulation

1. INTRODUCTION

In this paper we study short-term production planning in a multi-item factory with capacity constraints. The factory produces items that are shipped to a warehouse. The capacity of the factory can be shared freely between the different items, but items have to be produced in given fixed batches. Customers buy items from the warehouse on a daily basis. If their demand is not fulfilled, it is moved to the next day. During each day, we want to determine the production amounts for each of the product types. We are interested in the number of back-ordered items, i.e. how many times the demand cannot be fulfilled. We try to minimise that number, while keeping the stock size reasonable. The idea is to find solutions that are easy to implement and are computationally efficient and thus useful in practical production planning.

From the theoretical point of view, the problem is a dynamic lot-sizing problem with capacity constraints. Lot sizing has been studied thoroughly in the literature, and even without demand uncertainty, in a deterministic case, these kinds of problems are known to be hard. Hard problems can be solved optimally with MILP (Mixed Integer Linear Programming) models, but they usually require long solution times and so they are not practical for everyday use. For lot-sizing MILP models in industrial situations, see a review paper by Jans and Degraeve [1]. Uncertainty can be taken into account using robust optimisation, which tries to optimise the expected result, taking the whole distribution into account [2]. This type of robust optimisation is not practical for everyday use either.

In practice, operational solutions usually use base stock rules. The classical economic order quantity (EOQ) model finds the optimal solution in a static situation with a number of assumptions, e.g. production does not have any capacity limitations. In a dynamic stochastic situation, as in our case, statistical inventory models must be used. In a stochastic environment, the target stock level can be calculated using the service level. If the target stock is calculated using a 99% service level, in 99% of the demand situations we will have the product in stock. We also use this kind of base stock approach, but we consider a production capacity constraint, which complicates the situation.

The capacity constraint is the main difficulty in our setting. For the problem to be feasible, inventories must be stable, i.e. the capacity must be greater than the average demand [3]. On the other hand, if the capacity is substantially greater than the average demand, we can easily increase the stock level to the target level and the problem becomes trivial. We are interested in the situation where the average demand is near the capacity and the demand is occasionally greater than the capacity. This is typical in industrial situations.

As the capacity constraint restricts production, we should select what to produce. This can be done in a “Censored” way [4], i.e. we calculate the optimal production amounts and after that apply capacity constraints to the amounts. We calculate priorities for the products and produce the products that are more urgent than others. In the literature, some priority rules exist. A common way is to use the classical FCFS (First Come First Served) policy [5]. In some cases it minimises the variation in the waiting time. The LQ (Longest Queue) priority always produces the item with the greatest difference between the inventory level and the target level. It has been studied by Cohen [6] and Zhen and Zipkin [7]. It is optimal in a situation with identical products. There is also the PR (Priority) policy of Youssef et al. [8]. The PR policy prioritises the production of low-demand products so that stock levels can be reduced. The stock levels of low-demand items are set to zero so that they are actually make-to-order items. Although Youssef et al. compare only FCFS and PR, their approach looks similar to ours.

In this paper we use the stock control system to control the production. We test various priority rules and simulate them under different conditions. We find out that rules that take into account the whole demand distribution perform better than average-based ones.

2. THE PRODUCTION-INVENTORY SYSTEM

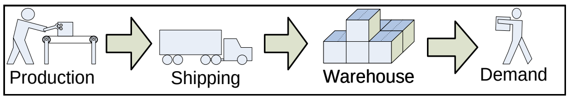

Fig. 1: Production-inventory system

2.1 The production-inventory system

We consider the production-inventory system that is shown in Figure 1. The factory produces multiple items with a common capacity restriction. The capacity can be shared freely between the different products, but products have to be produced in given batches. The products are shipped to the warehouse, which keeps inventory so that the demand can be fulfilled instantly. The demand is uncertain and it occurs only on weekdays. If a demand cannot be fulfilled, it is moved to the next day. During each day, we want to determine the production amounts for each of the products. We are interested in the number of back-ordered items, i.e. how many times the demand cannot be fulfilled. We try to minimise that number, while keeping the stock size reasonable.

2.2. Demand model

The average demand is known in the long run but daily demand varies. The variation depends on the average demand for the products, so that if a product has more demand it has relatively less variation. We use different gamma distributions to simulate the daily demand.

2.2. Base stock control system (BSCS)

Our solution for daily production can be described as follows. For each item, we calculate the target stock levels. We use target levels that are constructed using the demand distributions. After calculating the target levels, we consider those items whose stock levels are below their targets. For each of these items we calculate a priority value that indicates how urgent the production of the selected item is. The item with the largest priority is produced. This procedure is repeated until all the capacity is used or all items are at their target stock levels.

2.3 Target levels

The target level shows where the level of the inventory should be. If the inventory is less than the target level, we should produce until the inventory is again at the target level. Central features affecting desirable target stock sizes are the demand distribution and the production lead time. The target stock should also cover risks that arise from failures and material stockouts. We use target levels that exceed the demand over one lead time with a 99% probability. The intuition is that there is only a 1% probability of getting a back-order before new well-chosen supplies arrive at the warehouse.

2.4 Priorities

After calculating the target levels, we consider those items whose stock levels are below the target. For each of these items we calculate a priority value that indicates how urgent the production of the selected item is. If multiple products have the same priority, then the product that has a greater average demand is produced. The priority can be calculated by taking into account the current stock level, target stock level, average demand, and demand distribution. We study the following priority rules:

- AD – How long inventory lasts with average demand (i.e. stock/average demand)
- LQ – longest queue (i.e. stock – target stock)
- EB – expected back-ordered items

The AD priority takes the average demand into account and tries to produce the item that has the lowest storage level compared to the average demand. LQ produces the item whose stock level is farthest away from the target level. This priority is studied in the literature, but it does not take the demand into account in any way, and thus it is easy to use in practice. EB calculates the expected number of back-ordered items on the day when the current production shipment arrives at the warehouse. It should take into account the demand between the production time and the time of arrival. The idea of EB is shown in Figure 2. It uses the whole demand distribution and it is thus harder to calculate than AD and LQ.

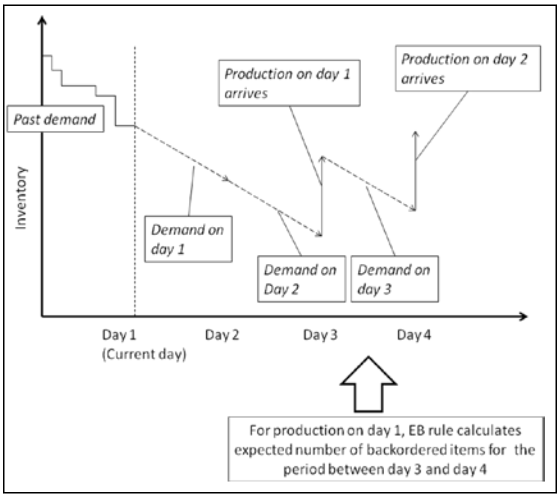

Fig. 2: The EB rule calculates the expected number of back-ordered items from the day the current production shipment arrives to the day when the next shipment arrives.

3. SIMULATION RESULTS

3.1. Experimental setup

The system described in the previous chapter is implemented in a simulator that is used to compare the different rules. By using the simulator, we examine capacity and back-ordered items. In the examples, we study the normal situation, where stock levels are at their target levels, and also the situation where stock levels are below their target levels. The situation under study deals with products with an average daily demand ranging from 0.2 to 18 items. The importance of all these products is the same, i.e. back-ordering one item with a demand of 0.2 items costs the same as back-ordering one item with a demand of 18 items. In practice, the cost of back-ordering an item that has a high average demand is expected to be greater than the cost of an item with a low demand. This is because customers typically have an idea of which are low-selling products and tolerate more backlogging for these. The production batch size for every product is 10 units. For the demands, we have three different distribution types: one for products with average demands from 0.2 to 1, one for products with average demands from 2 to 10, and one for average demands from 12 to 18. The distributions and their parameters are shown in Table 1.

Average	**Demand distribution**
0.2-1	Gamma distribution with shape parameter 0.25 and scale 2 scaled to the average demand
2-10	Gamma distribution with shape 3 and scale 2 scaled to the average demand
12-20	Gamma distribution with shape 10 and scale 1.4 scaled to the average demand

Table 1: Demand distributions in situation studied

The actual distributions are constructed by taking the corresponding distribution type and scaling it to fit to the actual average demand. This is done in this way because in the industrial application the exact distribution was, for practical reasons, harder to determine than the average demand estimation.

The target levels are set to a 99% service level for daily demand. Items arrive at the warehouse a day after their production and thus the target level is calculated for two days. The target levels are shown in Table 2.

Product	**Average demand**	**Target level (99%)**
LD1	0.2	4
LD2	0.4	8
LD3	0.6	12
LD4	0.8	17
LD5	1	21
MD1	2	9
MD2	4	18
MD3	6	27
MD4	8	36
MD5	10	45
HD1	12	39
HD2	14	46
HD3	16	52
HD4	18	59
HD5	20	65

Table 2: Average demands and target levels in the situation studied

3.2. Different capacities and different starting stock levels

We study the production and demands that were described above. We vary the capacity and starting stock levels. The capacity is initially set to be equal to the sum of the average demands of the products and then it is increased until it is twice as large as the average demand. Starting stock levels range from the situation of zero stocks to 100% of target stocks, with 20% increments. The priorities AD, LQ, and EB are tested.

Each of the different parameter value combinations is simulated with 10 different instances, where demands are randomly generated from the corresponding demand distributions. The same instances are used for all combinations. Each instance covers 50 days. We perform the simulations and calculate the average number of back-ordered items per day.

Figures 3, 4 and 5 show the behaviour of the system in those situations where stock levels are initially at 0%, 60% and 100% levels. The numbers of back-ordered items are shown as functions of production capacity with the different rules.

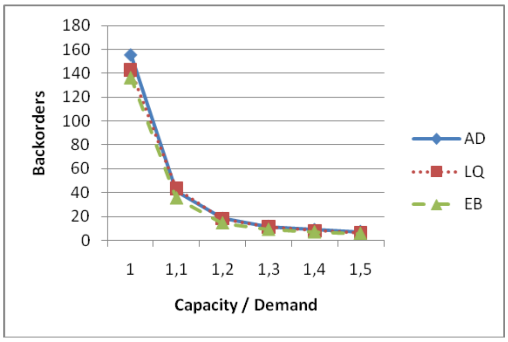

Fig. 3: Back orders when initial stock levels are at 0% of their target levels

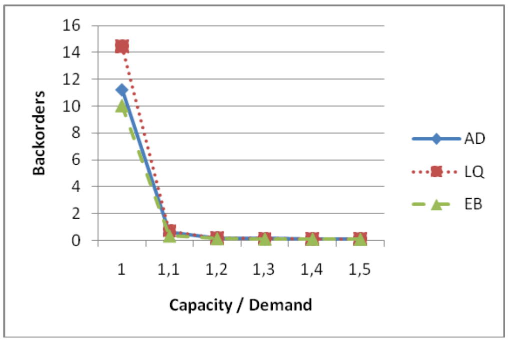

Fig. 4: Back orders when initial stock levels are at 60% of their target levels

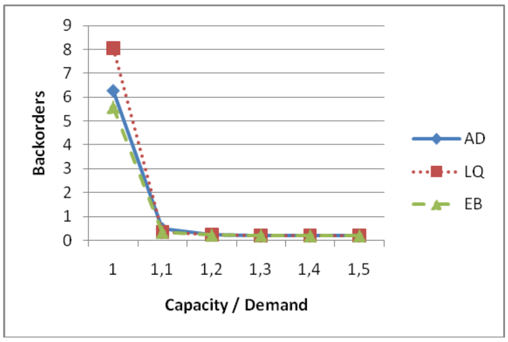

Fig. 5: Back orders when initial stock levels are at 100% of their target levels

4. DISCUSSION

In our example situation, the EB rule outperforms the other priority rules in most situations.

In Figure 3 we have a situation where the stocks are initially zero. In these situations, the differences between the performances of the priority rules are small. The LQ rule produces the product that has the highest target level, which is the product that has the greatest demand. The EB rule and AD rule produce the product which is not in stock.

In Figure 4 stock levels are initially at 60% of their target levels. In this case EB outperforms the other rules and AD gives nearly the same good result, but the LQ rule begins to perform the worst in the situation where capacity is the same than demand.

In Figure 5 the stock levels are initially close to their target levels. In these situations, the absolute differences in the results are not large. But again the EB rule outperforms other rules, and LQ rule gives the worst result.

The procedures have been tested in a real application with an industrial partner, with a similar situation and similar results.

5. CONCLUSION

In this paper, various priorities for updating stock levels are tested. The results of the simulations show that the EB rule, which takes into account the whole demand distribution, performs slightly better than the average-based rule AD. The LQ rule that is known in the literature does not perform well in updating the stock levels. It should be noted that these results are from a single, specific situation, where the demands range from products that have a nearly constant high daily demand to products that are bought occasionally. On the other hand, this kind of situation is common in multi-item production.

Although we got similar results with a real application, it is not yet known which other factors affect the behaviour of the system. It seems that, for example, the production batch size influences the results. Other factors that have an effect may be the demand distribution or relative difference between the lowest average demand and highest average demand. These issues need further research.

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