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IMPACT VIBRATION ABSORBER OF PENDULUM TYPE

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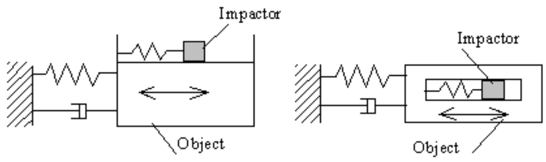
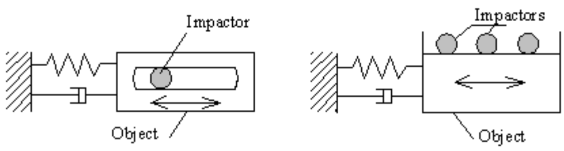
¶ **Abstract:** *In this work the impact vibration absorber of pendulum type is examined. It consists of pendulum with motion limiting stops attached to the vibrating system. Pendulum vibration absorbers are widely used in practice. The influence of pendulum parameters on the possibility of suppression of vibrations of the basic system under harmonic excitation is discussed in this study.*
Key words: vibration, absorber, amplitude, pendulum, frequency.

1. INTRODUCTION

Vibration is a repetitive, periodic or oscillatory response of mechanical system. Since most of machines and structures undergo some degree of vibrations, engineers have to consider the results of vibrations in the designing process [4], [8]. It is usually required to control the vibrations because it causes fatigue and failure of the vibrating elements and discomfort for the people. One of the most effective passive control methods is adding an impact vibration absorber (IVA) to the system under excitation [1], [2], [5]. IVA consists of impact mass which is placed on basic vibrating mass so, that periodically collides with it. The transfer of momentum to the impact mass from the main mass and dissipation of energy in every impact provide reduction in amplitude response of the main mass. IVA are fulfilled with one, two and more degrees of freedom; noncontrollable and regulated; with unilateral or with bilateral constraints. In accordance with structural type impact vibration absorbers may be spring (Fig.1), floating (Fig.2) and pendular (Fig.3).



a) outer b) inner
Fig.1.a-b. Spring impact absorbers



a) single –unit b) multi –unit
Fig.2.a-b. Floating impact absorbers

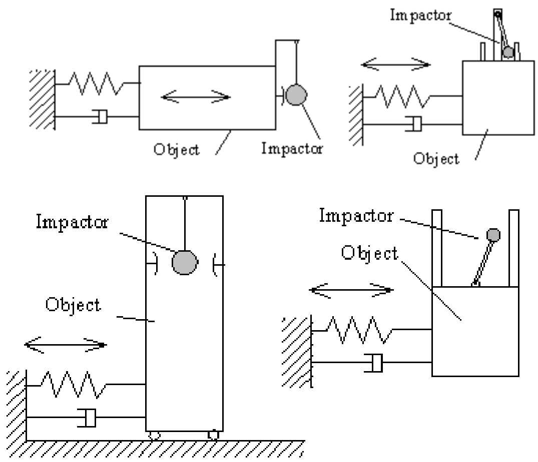


Fig.3. Pendulum impact absorbers

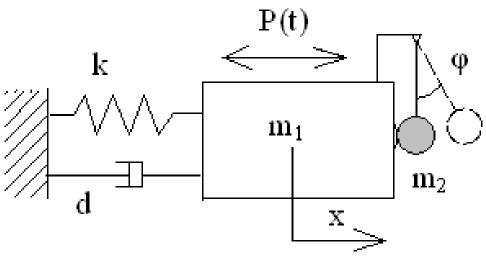
In this work the impact absorber of pendulum type is examined. It consists of pendulum with motion limiting stops attached to the vibrating system. Pendulum vibration absorbers are used in practice for decreasing of vibration level of different engineering structures: flue pipes, television towers, bridges, high-rise buildings, aerial masts, for shaft autobalancing and others [3], [6], [8].
The purpose of this research is to study the influence of parameters of the pendulum

on possibility of vibration suppression of the basic system under harmonic excitation, and the effect of the system parameters on system dynamics. This involved determination the effect of mass ratio, excitation amplitude, and clearance between impact stop walls. A pendulum with one and two impacts during the period is considered. Dependence of suppression ability of absorber on pendulum length, coefficient of restitution at impact, mass ratio of the basic system and pendulum, and gap size are found.

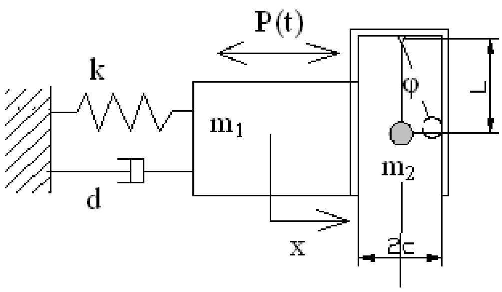
2. ANALITICAL MODEL OF ABSORBER

2.1 Mathematical model

In Fig.4 the models of single and double impact pendulum absorbers are presented.



a) single-impact absorber model



b) double impacts absorber model

Fig.4. Model of the pendulum impact absorber

Parameters of system:

m_1 – mass of the main body;
m_2 - mass of the damper;
$\mu = m_2/m_1$ - mass ratio;
d – inherent damping coefficient of the main system;
k – stiffness coefficient of main system;
r - coefficient of restitution of the velocity after impact;
λ - natural frequency of the main system;
ω – the frequency of pendulum;
l - length of pendulum;
c – size of gap;
α - max angle for two-impacts absorber, $\tan\alpha = c/l$;

The system is considered under harmonic excitation:

$$P(t) = P_0 \sin \Omega t ,$$

P_0 - amplitude of excitation force;
Ω – frequency of excitation.

2.2 The equations of motion of the system

Using Lagrange's equations the equation of motion of the examined system is derived:

$$\begin{cases} (m_1 + m_2)\ddot{x} + m_2 l\ddot{\varphi}\cos\varphi = -kx - d\dot{x} + \\ + m_2 l\dot{\varphi}^2 \sin\varphi + P_0 \sin\Omega t - S\sum_{R=0}^{\infty}\delta(t - RT); \\ m_2 l^2\ddot{\varphi} + m_2\ddot{x}l\cos\varphi = -m_2 gl\sin\varphi + \\ + S\sum_{R=0}^{\infty}\delta(t - RT) \end{cases} \quad (1)$$

where:
S - impact impulse,
$\delta(t - RT)$ – delta function,
T – period of collisions.

The stereomechanical theory of impact is used for impact impulse definition [7]:

$$S = (1 + r)\frac{m_1 m_2}{m_1 + m_2}(v_{01} - v_{02}) \quad (2)$$

where

v_{01} and v_{02} - velocity of main body and velocity of impactor just before impact.

The velocity of impactor consists of translational velocity and relative velocity:

$$v_{01} - v_{02} = v_{01} - (v_{01} + v_{2r}) = -l\dot{\varphi}_{01}, \quad (3)$$

here the angular velocity is pendulum velocity just before impact,

$$\dot{\varphi}_{01} = \dot{\varphi}(T) .$$

Taking into account (3) impact impulse may be represented as:

$$S = (1 + r)\frac{m_2}{1 + \mu}(-l\dot{\varphi}) . \quad (4)$$

After rearrangement of equation of system (1) and taking into account (4) system (1) may be written:

$$
\begin{cases}
\ddot{x}+\frac{b}{1+\mu}\dot{x}+\frac{\lambda^2}{1+\mu}x=\frac{\mu}{1+\mu}l\dot{\varphi}^2\sin\varphi-\\
-\frac{\mu}{1+\mu}l\ddot{\varphi}\cos\varphi+\frac{p_0}{1+\mu}\sin\Omega t+\\
+\frac{(1+r)\mu}{(1+\mu)^2}l\dot{\varphi}(T)\sum_{R=0}^{\infty}\delta(t-RT);\\
\ddot{\varphi}+\omega^2\sin\varphi=-\frac{\ddot{x}}{l}\cos\varphi-\\
-\frac{1+r}{1+\mu}\dot{\varphi}(T)\sum_{R=0}^{\infty}\delta(t-RT)
\end{cases}
\quad (5)
$$

where:

$$\lambda=\sqrt{\frac{k}{m_1}},\quad b=\frac{d}{m_1},\; p_0=\frac{P_0}{m_1},\; \omega=\sqrt{\frac{g}{l}}.$$

2.3 Analytical solution of the simplified equations of motion

For the simplified variant of equations of motion of the system – without taking into account dissipation and inertias forces:

$$
\begin{cases}
\ddot{x}+\frac{k}{m}x=\frac{P_0}{m}\sin\Omega t-\frac{S}{m}\sum_{R=0}^{\infty}\delta(t-RT)\\
\ddot{\varphi}+\omega^2\varphi=-\frac{\ddot{x}}{l}+\frac{S}{m_2 l}\sum_{R=0}^{\infty}\delta(t-RT),
\end{cases}
\quad (6)
$$

where $m=m_1+m_2$, with help of method of fitting an analytical solution is found [6]. Purely forced vibrations of the system and absorber for a time domain (0, T) between impacts under conditions of tuning:

$$\Omega T=2\pi; \qquad 2\omega=\Omega$$

for resonance condition $\lambda/(1+\mu)=\Omega$:

$$\tilde{x}(t)=\frac{p_0}{2\lambda^2}\cdot\left(-\frac{3}{2}\cos\frac{\lambda t}{\sqrt{1+\mu}}+\left(\frac{3\pi(1+2\mu)(1-r)}{4\mu(1+r)}-\pi+\frac{\lambda t}{\sqrt{1+\mu}}\right)\sin\frac{\lambda t}{\sqrt{1+\mu}}\right),$$

$$\tilde{\varphi}(t)=\frac{2p_0}{3l\lambda^2}\cdot\left(\frac{\pi(3-2\mu)}{2\mu}\sin\frac{\lambda t}{2\sqrt{1+\mu}}-\left(\frac{3\pi(1+2\mu)(1-r)}{4\mu(1+r)}-\pi+\frac{\lambda t}{\sqrt{1+\mu}}\right)\sin\frac{\lambda t}{\sqrt{1+\mu}}\right), \quad (7)$$

where the notations are as agreed above.

2.4 Numerical solution of equations of motion

In this work the numerical solution of system (5) was obtained with help of Euler method using the kinematics conditions – pre-impact and post- impact velocities of moving bodies if coefficient of restitution is known. The velocity of the main body v1 and velocity of impactor v_2 just after impact are:

$$v_1=v_{01}+l\dot{\varphi}_{01}\frac{\mu(1+r)}{1+\mu}. \quad (8)$$

$$v_2=v_{01}+l\dot{\varphi}_{01}\frac{\mu-r}{1+\mu}. \quad (9)$$

Algorithm of Euler's method for the sinle-impact damper, taking into account (8),(9):

$$t_{n+1}=t_n+\Delta t,$$

$$x_{n+1}=x_n+\dot{x}_n\Delta t$$

$$\varphi_{n+1}=(\varphi_n+\dot{\varphi}_n\Delta t)if(\varphi_n\geq 0,0,1)$$

$$\dot{x}_{n+1}=\dot{x}_n+\ddot{x}_n\Delta t+l\dot{\varphi}_n\frac{\mu(1+r)}{1+\mu}if(\varphi_n\leq 0,1,0)$$

$$\dot{\varphi}_{n+1}=\dot{\varphi}_n+\ddot{\varphi}_n\Delta t+\dot{\varphi}_n\frac{\mu-r}{1+\mu}if(\varphi_n\leq 0,1,0)$$

$$\ddot{x}_{n+1}=-\frac{b}{1+\mu}\dot{x}_n-\frac{\lambda^2}{1+\mu}x_n+\frac{p_0}{1+\mu}\sin\Omega t_n+\frac{\mu}{1+\mu}l\dot{\varphi}_n^{\,2}\sin\varphi_n-\frac{\mu}{1+\mu}l\ddot{\varphi}_n\cos\varphi_n$$

$$\ddot{\varphi}_{n+1}=-\dot{\varphi}^2\sin\varphi_n-\frac{\ddot{x}_n}{l}\cos\varphi_n$$

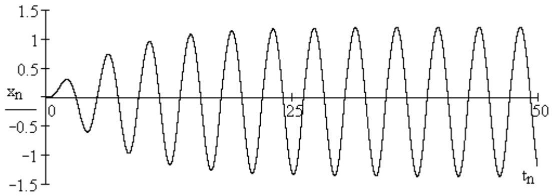
Euler method gives good results if time interval Δt is small. The equations of motion are solved numerically with help of Matcad program. The received results enable to analyze all parameters of motion of the system.

Examples of the solution of motion are presented below for single and two-impact absorbers.

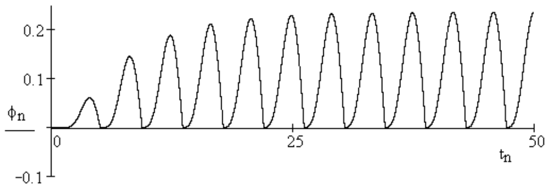
3. NUMERICAL EXAMPLE

For the numeral solution next value of parameters are accepted $\lambda=1.5$, $b=0.1$, $p_0=0.5$. Parameters values are chosen for civil engineering conditions. The structure is modeled as single-degree of freedom system, after adding the pendulum absorber it becomes two freedom degrees, the exiting force is harmonic.

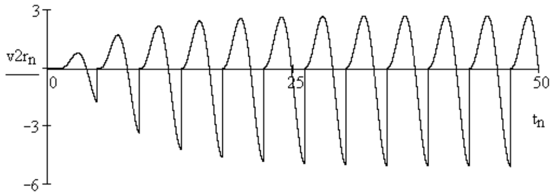
Parameters of motion of single-impact absorber are presented in Fig. 5 a-e, multi-impact absorbers - in Fig. 6 a-e, 7a-e.



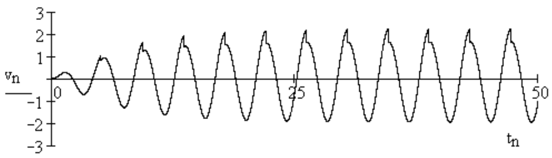
a) $x=x(t)$



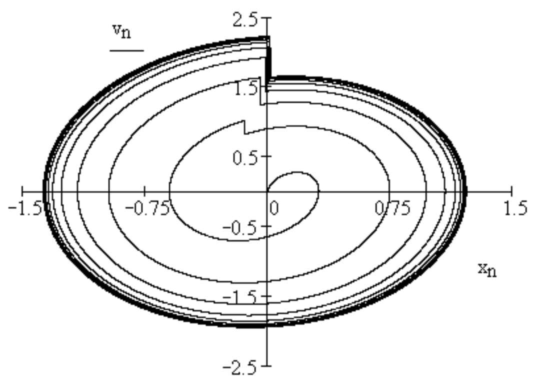
b) $\varphi=\varphi(t)$



c) $v_2r=v_2r\ (t)$



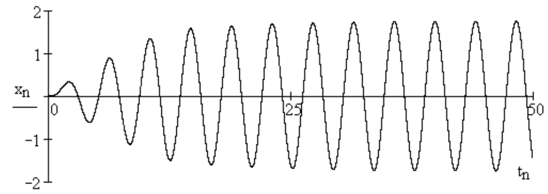
d) $v=v(t)$



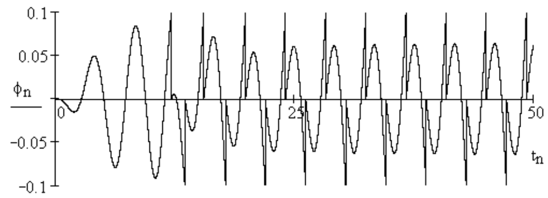
e) $v=v(x)$

Fig.5.a-e. Plots of dependence of motion parameters on time and phase map for one-impact absorber in case of: $\Omega=1.5$, $\lambda=1.5$, $\omega=0,75$, $\mu=0.04$, $e=0.6$

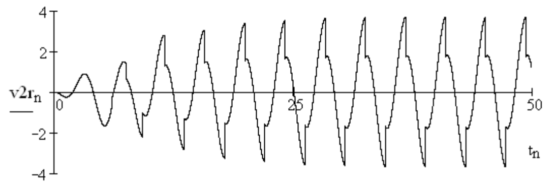
Plots in Fig.5-7: a) $x=x(t)$ - displacement of mass m_1, b) $\varphi=\varphi(t)$ - rotation angle of pendulum, c) $v_2r=v_2r\ (t)$ - relative velocity of mass m_2 , d) $v=v(t)$ - velocity of mass m_1, as functions of time t, e) $v=v(x)$ - velocity of mass m_1 as function of mass displacement x.



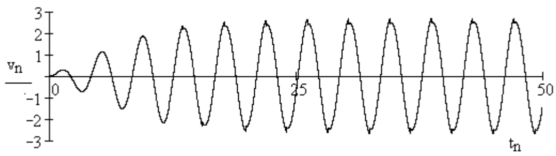
a) $x=x(t)$



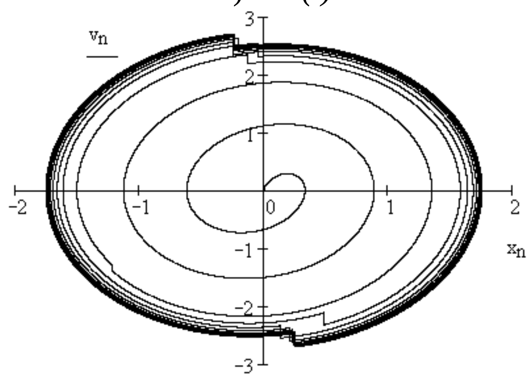
b) $\varphi=\varphi(t)$



c) $v_2 = v_2r\ (t)$



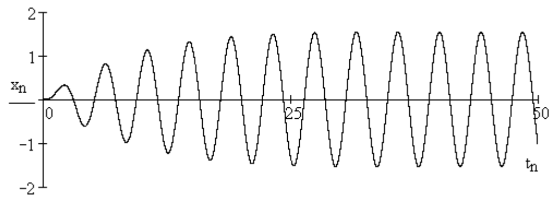
d) $v=v(t)$



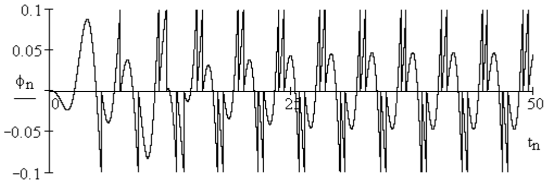
e) $v=v(x)$

Fig.6.a-e. Plots of dependence of motion parameters on time and phase map for double-impact absorber in case of: $\Omega=1.5$, $\lambda=1.5$, $\omega=0.75$, $\alpha=0.1$, $\mu=0.04$, $e=0.6$

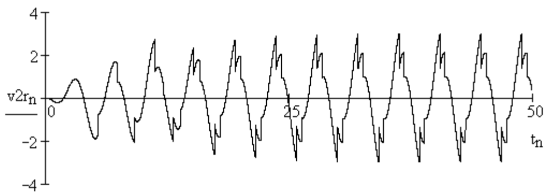
The absorber with parameters *ω=1.0 α=0.1* appears multi-impacts – it shows four impacts during period (Fig.7).



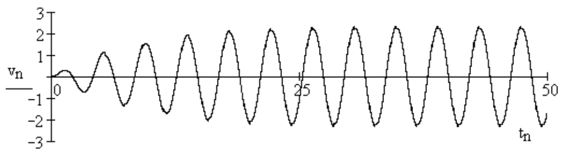
a) $x=x(t)$



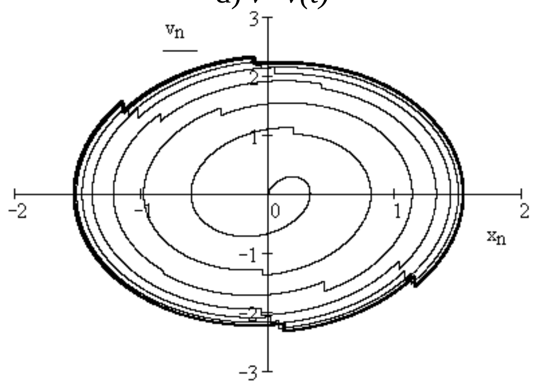
b) $\varphi = \varphi(t)$



c) $v_2r = v_2r\ (t)$



d) $v=v(t)$



e) $v=v(x)$

Fig.7.a-e. Plots of dependence of motion parameters on time and phase map for the case of *Ω=1.5, λ=1.5, ω=1.0, μ=0.04, α=0.1 e=0.6*

Plots of maximal amplitude A_{max} of main body in relation to exiting force frequency *Ω* for different pendulum frequences *ω* are presented in Fig.8-9, plots of A_{max} deprnding on mass ratio *μ* – in Fig.10, depending on angle α–in Fig.11.

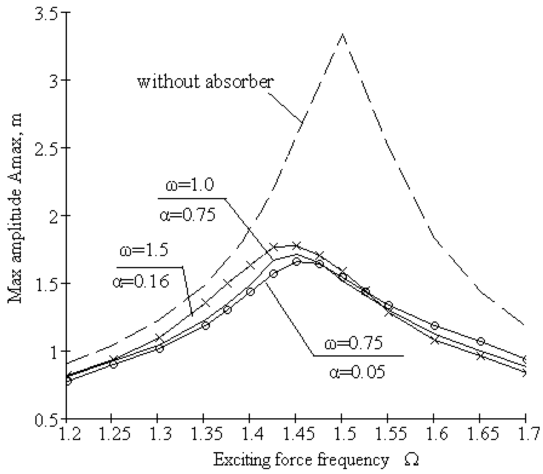


Fig.8. Maximal amplitude Amax in relation to exiting force frequency *Ω* for different multi-impacts pendulum frequencies and *α, μ=0.04, r =0.6*

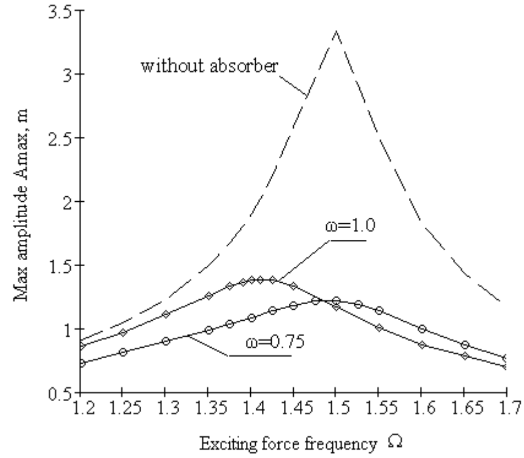


Fig.9. Maximal amplitude Amax in relation to exiting force frequencies *Ω* for single impact pendulum absorber (*μ=0.04, r=0.6*)

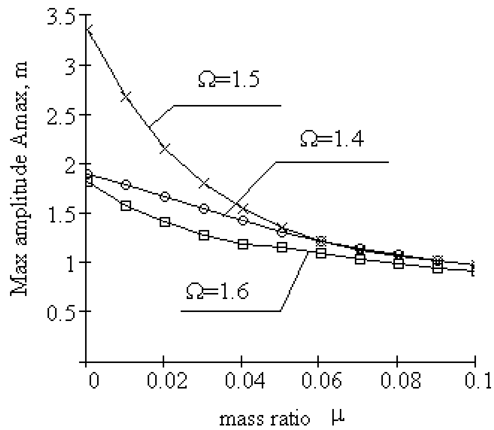


Fig.10. Maximal amplitude *Amax* depending on μ –ratio for multi-impact absorber *(ω=0.75, r=0.6, α=0.05)*

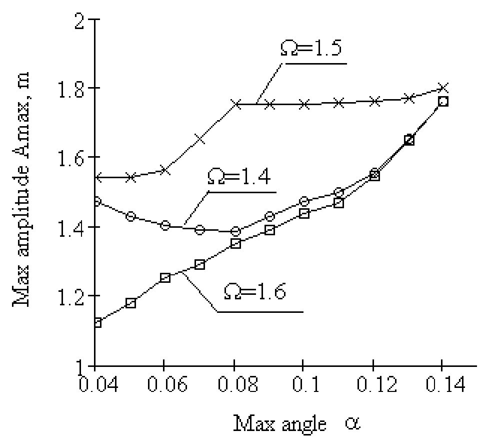


Fig.11. Maximal amplitude *Amax* depending on pendulum clearance angle α ($\omega=0.75$, $\mu=0.04$, $r=0.6$)

4. CONCLUSION

The differential equations of motion of the vibrating system are derived on the basis of Lagrange's equation of the second type.
The impacts in the system are described as impacts of perfectly rigid bodies taking into account the coefficient of restitution.
The equations of motion are solved numerically with help of Matcad program, using Euler's method. Numerical solution allows calculating not only the parameters of motion in the steady-state mode, but also in a transitional process. All parameters of transient motion and steady-state motion were defined, results were analyzed. Dependences of amplitude of vibrations are shown graphically on correlation of the masses, maximal of pendulum Amplitude in the graphs is shown maximal, instead of amplitude of the set motion.
For a one-impact absorber, adjusted on resonance frequency, attenuation ability is greater, but velocity of collisions is great, that can result in the damage of material.
In future it is necessary to take into account resilient properties of impact contacts using the dynamics conditions – to add the contact forces in impact contact point.

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