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MODIFICATION OF MODAL PROPERTIES OF MACHINING TOOL BODIES

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Abstract: *Many of the machining tools have the shapes corresponding to cantilever beam structures. During the machining process, the tools are subjected to the several exciting effects (cutting speeds, cutting forces, chip creation process a.o.) affecting machining process as well as the dynamics of MTW system (Machine-Tool-Workpiece). The vibration analysis of machining tool body is solved in this paper. The required modal properties of cantilever beam structure are obtained by structural modification of tool body using the reinforcing core. The effect of material properties and geometrical parameters of reinforcing core on natural frequencies of cantilever structure is presented.*
Key words: machining tool, beam vibration, reinforcing core, structural modification, natural frequency.

1. INTRODUCTION

Beam structural elements are one of the most important building elements of technical equipment and structures. Widely used applications of beam structures occur in technology production machines, where they may be used as machine tools - lathe tools, drills, boring bars, etc. During the technological process, the various excitation effects affect the machine tools. Cutting speeds, cutting forces, chip-making manner, the stiffness of MTW (machine-tool-workpiece) system are predominant effects affecting the dynamics of MTW system and also influencing the machining process (roughness of the machined surface, tool wear, tool or workpiece damage, noise generated by the machining process, etc.).

Based on the assessment of individual subsystems, the machine tools belong to the most critical members of the MTW system. The most serious problems in the machining process occur when the frequencies of periodic changes of the significant effects of machining process are close to the values of the tool natural frequencies. The resonant condition of machine tool occurs, which significantly affects the functionality of the tool and also the quality of the machining process. The modal properties (natural frequencies and mode shapes) depend on the geometrical parameters and material properties of the tool body. The tool bodies are usually made of homogeneous material and, in many cases do not have the required dynamic properties. It is therefore necessary to modify the tool to satisfy the requirements of elimination of undesirable dynamic effects. It has to be noted, that in accordance with the expected requirements, the layered composite structures can be applied, which are represented by sandwich or laminated beams. However, in many cases, the applications of the layered structures to the machining tool bodies can have design as well as technological restrictions (e.g. the required dynamical properties of layered beam structure are achieved mainly for uniaxial loading effect, delamination of structural layers, chemical instability of the layers material subjected to contact with coolant a.o.). One way how to modify the modal properties is the structural modification of the tool body by the reinforcing core. The manner of creation of the body tool presented in this paper is focused on obtaining of the required dynamical properties of the cantilever beam structure by inserting the reinforcing core into beam. The technique of the dynamic properties modification of beam composite structure is based on changes of material properties and geometrical dimensions of beam core. Generally, this arrangement of the beam structure makes it possible to create a light-weight structure consisting from closed outer profile (load supporting part of the profile) with inner space (core) of the profile filled by material of lower rigidity (e.g. aluminium foam). Contrary, the core can be used as reinforcing part of composite beam structure, i.e. the core is load supporting part of the profile.

2. FORMULATION OF THE PROBLEM

The modal properties of cantilever beam structural models with uniform circular or rectangular cross sections are analysed in this paper. The cross section of beam consists of a basic shape of profile (circle or rectangle) and centred reinforcing core with uniform circular cross section (Fig.1).

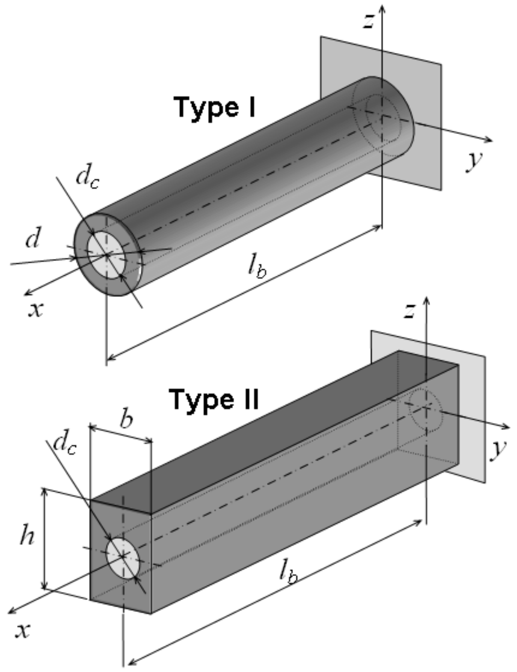


Fig. 1. Models of cantilever beams

2.1 Mathematical model of cantilever beam without reinforcing core

The equation of motion for the free bending vibration of Euler-Bernoulli beam with homogeneous and constant cross-section area [1], [4] is expressed in the form

$$\rho S \frac{\partial^2 w(x,t)}{\partial t^2} + EJ \frac{\partial^4 w(x,t)}{\partial x^4} = 0, \qquad (1)$$

where$S$ is the cross section area,EJ is the beam bending stiffness, J is the quadratic moment of the beam cross section, w is the beam deflection, ρ is the density and E is the Young´s modulus of beam material.

The solution of equation (1) is in the form

$$w(x,t) = W(x)e^{i\omega t}, \qquad (2)$$

and after introducing parameters:

• non-dimensional beam deflection

$$\overline{W}(\xi) = \frac{W(x)}{l_b}, \qquad (3)$$

• non-dimensional coordinate

$$\xi = \frac{x}{l_b}, \quad \text{for} \quad \xi \in \langle 0.0; 1.0 \rangle, \qquad (4)$$

the partial differential equation (1) are transformed into ordinary differential equation of the fourth order

$$\overline{W}^{IV}(\xi) - \beta^4 \overline{W}(\xi) = 0. \qquad (5)$$

where the frequency parameter has the form

$$\beta = \sqrt[4]{\omega^2 l_b \frac{\rho S}{EJ}}. \qquad (6)$$

The solution of (5) has the form

$$\begin{aligned} \overline{W}(\xi) &= A\sin\beta\xi + B\cos\beta\xi \\ &\quad + C\sinh\beta\xi + D\cosh\beta\xi, \end{aligned} \qquad (7)$$

where A, B, C, D are integration constants.

The boundary conditions [3] for cantilever beam (Fig. 1.) are expressed by

$$\overline{W}(0) = \overline{W}'(0) = \overline{W}''(1) = \overline{W}'''(1) = 0. \qquad (8)$$

After substituting (7) into the boundary conditions (8), the frequency equation is

$$1 + \cos\beta\cosh\beta = 0 \;\Rightarrow\; \beta_k \; (k = 1,2,\ldots) \qquad (9)$$

and the first four roots following from solution of equation (9) [1] are

$$\begin{aligned} \beta_1 &= 1.8751, & \beta_2 &= 4.6941, \\ \beta_3 &= 7.8548, & \beta_4 &= 10.9955. \end{aligned} \qquad (10)$$

Then, the natural angular frequency for the geometric parameters and material properties of the beam can be expressed in the form

$$\omega_k = \left(\frac{\beta_k}{l_b}\right)^2 \sqrt{\frac{EJ}{\rho S}}. \qquad (11)$$

2.2 Mathematical model of cantilever beam with reinforcing core

Next, we consider the cantilever beam with uniform cross section of basic beam profile having inserted reinforcing core with uniform circular cross section. The term "reinforcing" core is not the correct notion in the entire range of possible states of the core, since the reinforcing effect of the core is generated only when the structural parameters of the core raise the stiffness properties of the basic beam. In formulating the mathematical model of the beam structure modified by reinforcing core, the following assumptions are considered:

- reinforcing core is symmetrical with respect to the longitudinal beam axis (x-axis),
- beam cross section of the is perpendicular to the neutral axis x,
- beam cross section before and during strain is assumed as planar,
- isotropic and homogeneous material properties of structural parts of the beam,
- at the interface of structural parts of the beam the perfect adhesion is supposed.

The equation of motion for free bending vibration of the beam with reinforcing core has the following form

$$[\rho S+(\rho-\rho_c)S_c]\frac{\partial^2 w(x,t)}{\partial t^2} + [EJ+(E-E_c)J_c]\frac{\partial^4 w(x,t)}{\partial x^4}=0, \qquad (12)$$

where ρ_c is the density, S_c is the area of core cross section, E_c is the Young's modulus, J_c is the quadratic moment of the reinforcing core cross section.

By using the relations (2)-(4) in equation (12), the equation (5) is valid and frequency parameter β of modified beam structure is expressed by

$$\beta^4=\omega_M^2 l_b \frac{\rho S(1+\Delta_{\rho S})}{EJ(1+\Delta_{EJ})}, \qquad (13)$$

where ω_M is natural angular frequency of modified beam structure.

Solution of the equation (12) is expressed [3] in the form (7). For the boundary conditions (8) the first four roots β_k ($k = 1 \div 4$) are the same as it is presented in (10).

The non-dimensional variables defining changes in structural parameters are used:

- non-dimensional mass modification

$$\Delta_{\rho S}=(\kappa_\rho-1)\kappa_S, \qquad (14)$$

- non-dimensional stiffness modification

$$\Delta_{EJ}=(\kappa_E-1)\kappa_J, \qquad (15)$$

- non-dimensional density

$$\kappa_\rho=\frac{\rho_c}{\rho}, \qquad (16)$$

- non-dimensional Young's modulus

$$\kappa_E=\frac{E_c}{E}, \qquad (17)$$

- non-dimensional cross-section

$$\kappa_S=\frac{S_c}{S}, \qquad (18)$$

- non-dimensional quadratic moment

$$\kappa_J=\frac{J_c}{J}, \qquad (19)$$

- non-dimensional ratio of dominant cross-section dimensions (circular cross-section $D = d$; rectangular cross-section $D = h$)

$$\kappa_d=\frac{d_c}{D}. \qquad (20)$$

From (11) and (13), the natural angular frequency of the modified beam is

$$\omega_{M,k}=\omega_k\sqrt{\frac{1+\Delta_{EJ}}{1+\Delta_{\rho S}}}. \qquad (21)$$

It follows from (21) that for the modification of natural frequencies the modification function can be formulated in the form

$$f_M(\Delta_{\rho S},\Delta_{EJ})=\sqrt{\frac{1+\Delta_{EJ}}{1+\Delta_{\rho S}}}, \qquad (22)$$

where ω_k is k^{th} natural angular frequency of the homogeneous beam, $\omega_{M,k}$ is k^{th} natural angular frequency of the beam modified by the core.

The modification function $f_M(\Delta_{\rho S}, \Delta_{EJ})$ represents the change of the natural frequency value caused by changes in core structural parameters (density, core diameter, etc.).

The relative sensitivity S is used to analyze and assess the structural parameters influence on the modification function f_M, i.e.

$$S[f_M|_{p_j}]=\frac{p_j}{f_M}\frac{\partial f_M}{\partial p_j}, \qquad (23)$$

where p_i is the some of the non-dimensional structural parameters (14)-(20).

3. NUMERICAL RESULTS

The numerical analyses are performed for two cases of beam cross sections:

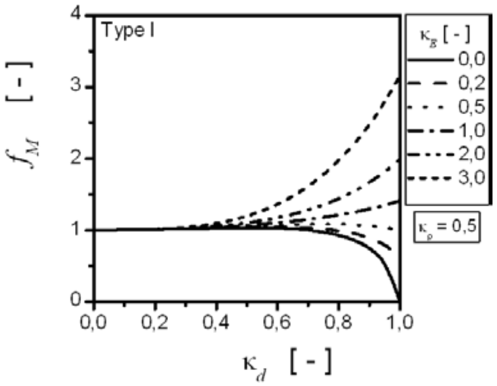
- Type I: circular cross-section of the beam *vs*. circular cross-section of core ($D = d$);

$$\kappa_d = \frac{d_c}{d}. \qquad (24)$$

- Type II: circular cross-section of the beam *vs*. circular cross-section of core ($D = h$);

$$\kappa_d = \frac{d_c}{h}. \qquad (25)$$

a)



b)

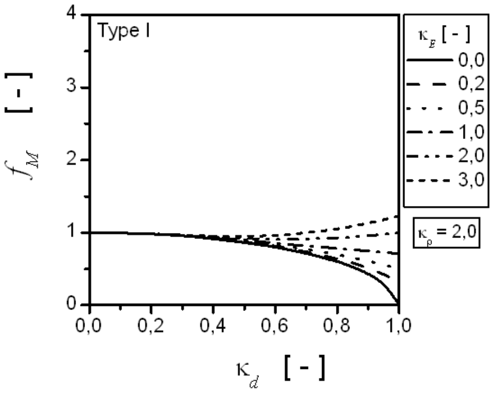
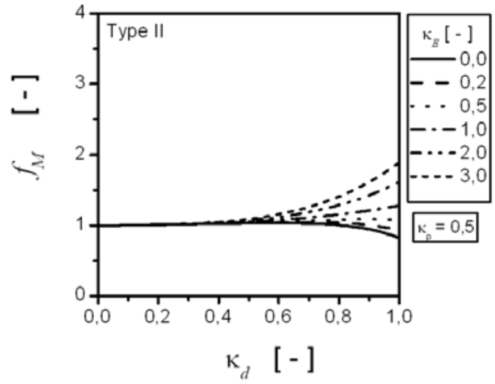


Fig. 2. Modification function f_M (Type I) *vs*. κ_d, parameters κ_E: a) $\kappa_\rho = 0.5$; b) $\kappa_\rho = 2.0$.

Dependencies of the modification function f_M on non-dimensional ratio κ_d for different non-dimensional Young's modulus κ_E and for constant of non-dimensional density κ_ρ ($\kappa_\rho = 0.5$; $\kappa_\rho = 2.0$) are shown in Fig.2 (Type I) and Fig.3 (Type II). Dependencies of the modification function f_M on non-dimensional ratio κ_d for different values of non-dimensional density κ_ρ and for onstant parameters κ_E ($\kappa_E = 0.5$; $\kappa_E = 2.0$) are shown in Fig. 4 (Type I) and Fig.5 (Type II).

a)



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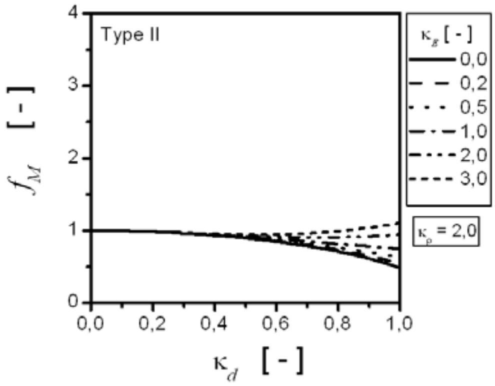
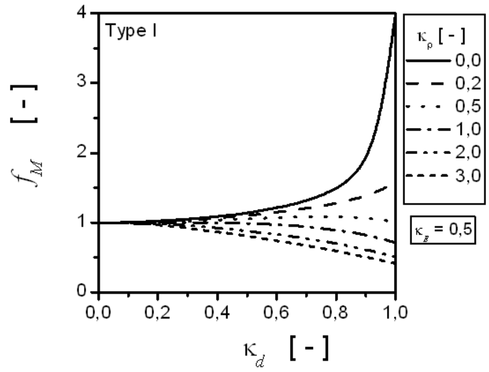


Fig. 3. Modification function f_M (Type II) *vs*. κ_d, parameters κ_E: a) $\kappa_\rho = 0.5$; b) $\kappa_\rho = 2.0$.

a)



b)

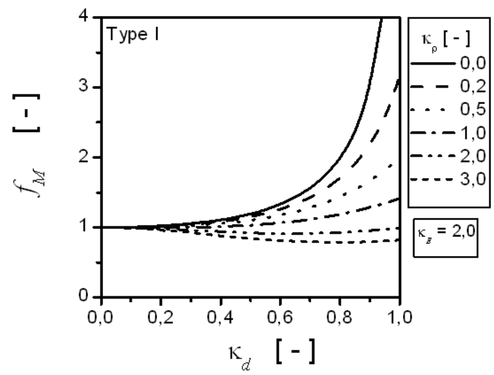


Fig. 4. Modification function f_M (Type I) *vs*. κ_d, parameters κ_ρ: a) $\kappa_E = 0.5$; b) $\kappa_E = 2.0$.

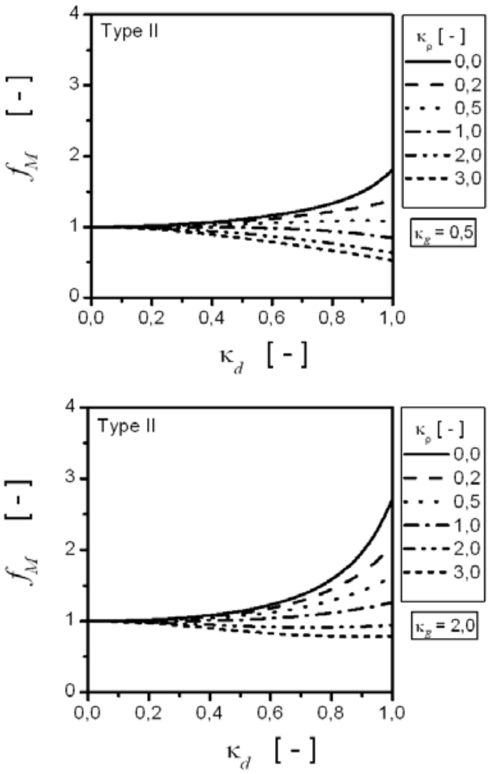


Fig. 5. Modification function f_M (Type II) *vs*. κ_d, parameters κ_ρ: a) $\kappa_E = 0.5$; b) $\kappa_E = 2.0$.

Dependencies of the modification function f_M on the ratio of dominant dimensions of the cross-section κ_d for parameters κ_E, resp. κ_ρ are presented in Fig. 2 - Fig. 5. The influence of changes in structural parameters of the reinforcing core on the modification function f_M can be better expressed in the dependence on the non-dimensional mass modification $\Delta_{\rho S}$ (14), or on the non-dimensional stiffness modification Δ_{EJ} (15). The dependence of the function f_M on the non-dimensional mass modification $\Delta_{\rho S}$, for different parameters of stiffness modification Δ_{EJ}, is shown in Fig. 6. The influence of non-dimensional stiffness modification Δ_{EJ} on the modification function f_M, for the parameters of the mass modification $\Delta_{\rho S}$, is shown in Fig. 7. As it follows from the assessment of the above mentioned dependencies, the growth of the mass modification value $\Delta_{\rho S}$ causes the decrease of modification function. Vice versa, if the value of stiffness modification Δ_{EJ} is increasing, the modification function has an upward trend. These statements are unambiguously confirmed by the results obtained by sensitivity analyses.

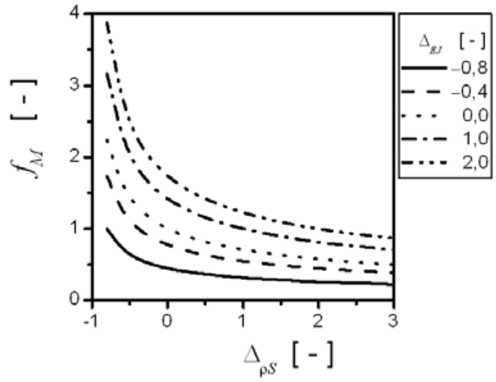


Fig. 6. Modification function f_M *vs*. non-dimensional mass modification $\Delta_{\rho S}$.

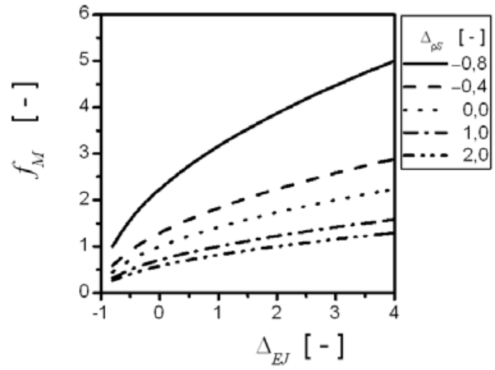


Fig. 7. Modification function f_M *vs*. non-dimensional stiffness modification Δ_{EJ}.

The relative sensitivity $S[f_M \,|\, \Delta_{\rho S}]$ of the modification function f_M (Fig. 8) is significantly decreasing for the value of the mass modification $\Delta_{\rho S}$ at interval $(-1.0; 0.0\rangle$, in which the weight of beam structure is being decreased. For the value $\Delta_{\rho S} = 0.0$, the beam structure in terms of density layout is homogeneous. At interval $\Delta_{\rho S} \in \langle 0.0;\infty)$, the sensitivity of modification function $S[f_M \,|\, \Delta_{\rho S}]$ also shows a decrease, but is not as marked as at the interval $\Delta_{\rho S} \in (-1.0;\ 0.0\rangle$. Unlike the sensitivity of modification function $S[f_M \,|\, \Delta_{\rho S}]$, the growth of sensitivity function $S[f_M \,|\, \Delta_{EJ}]$ (Fig. 9) occurs in the case of sensitivity analysis of modification function f_M in dependence on the stiffness modification Δ_{EJ}. At the interval $\Delta_{EJ} \in (-1.0;\ 0.0\rangle$, this growth is relatively considerable and continues also at interval $\Delta_{EJ} \in \langle 0.0;\ \infty)$, but with much lower gradient.

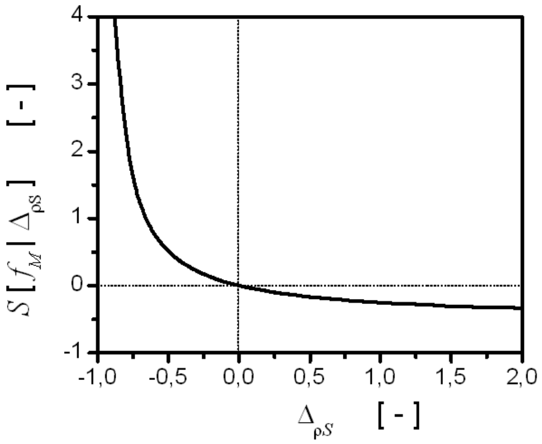


Fig. 8. Relative sensitivity of the function f_M vs. non-dimensional mass modification $\Delta_{\rho S}$.

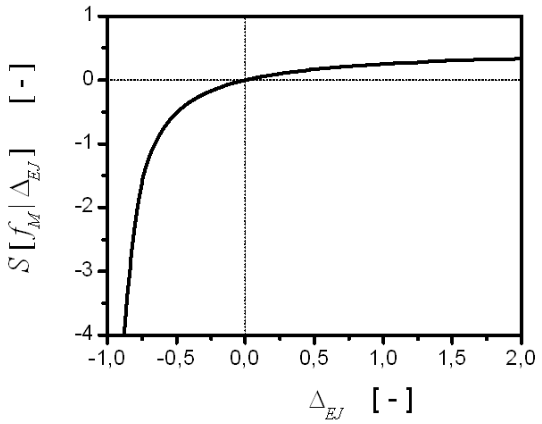


Fig. 9. Relative sensitivity of the function f_M vs. non-dimensional mass modification $\Delta_{\rho S}$.

4. CONCLUSIONS

The main aim of this paper was to modify the natural frequencies of the cantilever beam structure representing the machining tool body. Structural modification of the beam structures by structural parameters of the reinforcing beam core provides a relatively effective way of modification of their modal properties. Based on the formulation and analyses presented in this study, the modification function considering all the relevant structural changes of parameters is derived.

The natural angular frequency of the modified beam structure is then determined from the relation

$$\omega_{M,k} = f_M(\Delta_{\rho S}, \Delta_{EJ})\omega_k , \qquad (26)$$

for $i = 1, 2, ..., \infty$.

For the defined non-dimensional mass and stiffness modifications, the modification function $f_M(\Delta_{\rho S}, \Delta_{EJ})$ has the same value for all the natural angular frequencies ω_k ($k = 1, 2, ..., \infty$). The required natural frequency value is determined by using a suitable combination of the non-dimensional modification parameters $\Delta_{\rho S}$ and Δ_{EJ}. The results obtained confirm that this manner of the structural modification of the beam offers very effective tool to modification of dynamical properties or dynamical tuning for similar beam structures.

5. ACKNOWLEDGEMENT

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