7th International DAAAM Baltic Conference
"INDUSTRIAL ENGINEERING
22-24 April 2010, Tallinn, Estonia

INFLUENCE OF LOCAL METASTABILITY IN THE WORKABLE MATERIAL ON CONTROL OF THE CUTTING PROCESS

Maksarov, V.V. & Olt, J.

Abstract: *Hereby we propose one of the most effective methods, which enable to reliably control the cutting process of the hard-to-work materials with the induced preliminary local physical action (LPA), which leads to local structural metastability on the external surface of the cut off layer of the workable material, developing according to specific laws.*
Keywords: machining, cutting, local action, chip formation

1. INTRODUCTION

The study into the potential to control the rheological parameters in the process of cutting is based on the following positions, reflecting the long-term data on the dynamic behavior of technological systems involved in mechanical processing:

1) the nonlinearities, characteristic of a technological system, are not manifested essentially as the limiting factor of fluctuation level in the initial limited interval when the conditions for small deviations from steady state are retained;
2) practical use of the regime of auto-oscillations is permissible for expanding the technological possibilities of a system in the initial limited time interval $\boldsymbol{t_a} \ll \boldsymbol{\tau}$ (where $\boldsymbol{\tau}$ - the time of the practical establishment of the amplitude) when $A_i \leq [A_i]$, where $[A_i]$ - the tolerance level of the fluctuations;
3) establishment of auto-oscillations upon transfer of the system beyond the region of stability and approximation to a limiting cycle is characterized by the amplitudes of generalized coordinates of inadmissible value, which excludes the possibility of their practical use as operating conditions;
4) in the initial interval the solution to a nonlinear system of equations beyond the region of stability can be approximated to the solution of the linearized system of Lyapunov.

2. MATERIALS AND METHODS

On the basis of the above given positions for the analysis of the influence of local metastability in the workable material on the process of chip formation and the dynamic properties of the technological system the following assumptions were made [[1]]:

1) behavior of the technological system of mechanical processing remains unchanged beyond the region of stability and is determined on the basis of the linearized equations within the framework of the assumption about small motions;
2) for the description of increasing fluctuations in the system we disregard variation in the time lag of forces, and also the damping influence of tangential outline along the y-axis in connection with the small fluctuations in this direction, which is not substantially reflected in the thickness of the cut off layer and in the disturbances in the system.

3. THEORETICAL RESULTS

When applying the above made assumptions to the normal outline *x* the equation of motion for the technological system of mechanical processing takes the form [2]

$$m_x\ddot{x}+(b_x+\beta_3)\cdot\dot{x}+(c_x+c_3)\cdot x=F(x,\dot{x}), \quad (1)$$

where m_x - the inertia parameter; b_x - the parameter of dissipation of energy; c_x - stiffness constant; β_3, c_3 - the quasi-resilient and dissipative coefficients, which reflect the process of chip formation; $F(x, \dot{x})$ - the cutting force.

Machining with the induced preliminary local physical action (LPA) leads to the periodic alternation in the mechanical properties of the workable material and, therefore, also in all rheological parameters in the zone of chip formation from $G_1\{c_1,c_2,\beta_2,c_3,\beta_3\}$ to $G_2\{c'_1,c'_2,\beta'_2,c'_3,\beta'_3\}$. The most significant parameter is the quasi-resilient coefficient c_2, which reflects the processes, proceeding in the zone of plastic deformation of the cut off layer [3]. Changes in the rheological parameters result in the periodic formation of two oscillatory systems, which differ in terms of parameter c_2 in the modulation range of $2\mu = c_2 - c'_2$.

The diagram of the suppression of increasing fluctuations in the process of cutting is given in Fig. 1. The unstable system present in mechanical processing leads to the appearance of auto-oscillations with the increasing amplitude of up to the maximum permissible value A_{x1}, determined by the technological requirements set for machining the component. At the moment of time t_1 the fitting of the cutting wedge of tool into the LPA zone is performed, and the system accomplishes a passage from level G_1 of rheological properties onto level G_2. Meanwhile, the change from the quasi-resilient parameter c_2 in the interval T_p, which reflects the usual process of cutting, to the parameter c'_2 in the interval T_m of the LPA zone leads to the suppression of the increasing fluctuations. We define the steady boundary state of a dynamic model as such state, whereby in each period T the variable amplitude A_{x1} reaches the maximum permissible value A_{x2}.

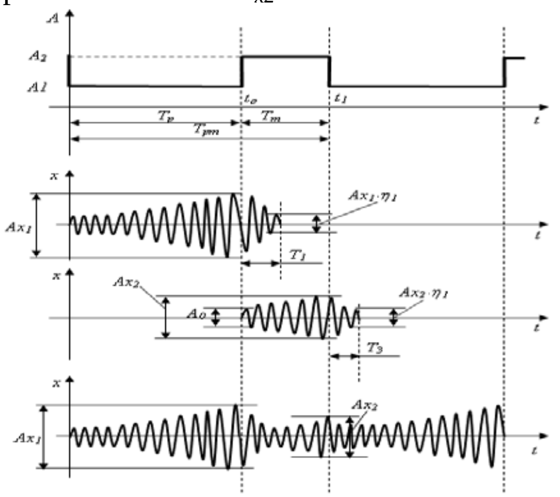

Fig. 1. Diagram of the suppression of increasing fluctuations in the process of cutting with the periodic alternation in rheological parameters of the workable material.

For the solution of the problem by the limitation of the amplitude of auto-oscillations to the value A_{x1} it is necessary to perform a number of conversions. Let us accept in the equation (1) the values

$$2h=\frac{b_x+\beta_3}{m_x};\quad \omega_0^2=\frac{c_x+c_3}{m_x};\quad F=\frac{F(x,\dot{x})}{m_x},$$

then we obtain equation (1) in the following form

$$\ddot{x}+2h\dot{x}+\omega_0^2x=F, \quad (2)$$

where F – the force, which reflects the working process, which takes place in the zone of cutting. As a result of the change in the rheological properties, the previous system begins to accomplish the free dying oscillations, which can be approximately described by the equation

$$\ddot{x}+2h\dot{x}+\omega_0^2x=0. \quad (3)$$

The general solution of this equation takes the well-known form

$$x_1=e^{-h\cdot t}\left(C_1\cdot\sin\omega\cdot t+C_2\cdot\cos\omega\cdot t\right) \quad (4)$$

where $h=\frac{b_x+\beta_3}{2m_x}$; $\omega=\sqrt{\omega_0^2+h^2}$; $\omega_0=\sqrt{\frac{c_x+c_3}{m_x}}$;

C_1 and C_2 – the constants, determined in accordance with the initial conditions are determined from the initial conditions $x(0) = x_0;\ \dot{x}(0) = \dot{x}_0$ in the form

$$C_1 = \frac{\dot{x}_0 + hx_0}{\omega};\quad C_2 = x_0.$$

Converting the solution of equation (4), we obtain

$$x_1 = Ae^{-h\cdot t}\cdot\sin\left(\omega_0 t + a_c\right), \tag{5}$$

where $A = \sqrt{\left[\frac{\left(\dot{x}_0 + hx_0\right)^2}{\omega_0^2 - h^2} + x_0^2\right]};$

$$a_c = \arctan\frac{x_0\sqrt{\omega_0^2 - h^2}}{\dot{x}_0 + hx_0}.$$

Thus, the motion is that of dying oscillations with a constant frequency and diminishing amplitude. The envelopes of the process of damping are determined by the function

$$A = \pm A_0 e^{-ht_c}, \tag{6}$$

where A_0 – the initial ordinate of the envelope; t_c – the current time.

We consider the process of damping fluctuations ended, if their amplitude falls within the range of up to five percent of the initial level, i.e. $x_1 = \eta_1\cdot A_{1x}$, where $\eta_1 = 0{,}05$.

$$A_{1x}\cdot\eta_1 = A_{1x}\cdot e^{-hT_1}, \tag{7}$$

then $e^{hT_1} = \frac{1}{\eta_1}$ and $hT_1 = \ln\frac{1}{\eta_1}$.

Time of delay of fluctuations during the period of cutting T_1 in the source material and during the period T_2 in the LPA zone are determined by the expressions

$$T_1 = \frac{2m_x}{b_x + \beta_3}\cdot\ln\frac{1}{\eta_1}; \tag{8}$$

$$T_2 = \frac{2m_x}{b_x + \beta_3'}\cdot\ln\frac{1}{\eta_1}. \tag{9}$$

In accordance with the existing classification, the fluctuations, which are supported due to the chronological variation in the parameters of the system, relate to the parametric fluctuations. For the analysis of stability of the periodic motions in the system we have used a linear differential equation with periodic coefficients, the general form of which can be represented as [4]:

$$A(t)\frac{d^2x}{dt^2} + B(t)\frac{dx}{dt} + C(t)x = 0, \tag{10}$$

where all coefficients of the equation are periodic functions of time with the period T:

$$A(t+T) = A(t). \tag{11}$$

The coefficients of the parametrically excitable system are, as a rule, assigned by two parameters: the frequency of excitation $\nu = \frac{2\cdot\pi}{T}$ and the modulation factor μ, which characterize the intensity of parametric action (depth of modulation of the parameter c_2).

With the use of a method of preliminary LPA on the metal of the cut off layer the frequency of excitation will be determined by the period of local action, while the depth of modulation will depend on the difference in the level of the mechanical properties (the quasi-resilient parameter c_2) of the workable material in the basic zone and in the zone of local action.

The equation of the fluctuations of a dissipative system with one degree of freedom is reduced to the form

$$\ddot{x} + 2\varepsilon\dot{x} + \omega_o^2\left[1 + 2\mu\Phi(t)\right]x = 0, \tag{12}$$

where $\Phi(t)$ - the excitation; T - the periodic function; μ – the depth of modulation.

With $\varepsilon = 0$ from the equation (12) the Mathieu-Hill equation is obtained

$$\ddot{x} + \omega_o^2\left[1 + 2\mu\Phi(t)\right]x' = 0. \tag{13}$$

If $\varepsilon \neq 0$, the equation (12) is reduced to form (13) by the substitution

$$q(t) = e^{-\varepsilon t}u(t). \tag{14}$$

Function $U(t)$ satisfies the equation

$$\frac{d^2u}{dt^2} + \omega_o^2\left[1 - \frac{\varepsilon^2}{\omega_o^2} + 2\mu\Phi(t)\right]u = 0. \tag{15}$$

In the absence of friction and the piecewise constant function of switching differential equation (2) takes the form (Meissner's equation)

$$\ddot{x} + \omega_o^2\left(1 \pm \mu\right)x = 0, \tag{16}$$

where $\mu = \frac{\Delta\omega_o^2}{\omega_o^2}$.

Taking into account the fact that during each period $\frac{T}{2} = \frac{\pi}{\omega}$ equation (16) has constant coefficients, it is possible to use the method of fitting [4]. Thus, the differential equation (16) takes the form

$$\ddot{x}_1 + \omega_o^2 (1+\mu) x_1 = 0, \text{ where } 0 < t < \frac{T}{2} \quad (17)$$

$$\ddot{x}_2 + \omega_o^2 (1-\mu) x_2 = 0, \text{ where } (\frac{T}{2} < t < T). \quad (18)$$

The solutions of differential equations (17) and (18), respectively, take the form

$$x_1 = C_1 \sin\omega_1 \cdot t + D_1 \cos\omega_1 \cdot t; \quad (19)$$

$$x_2 = C_2 \sin\omega_2 \cdot t + D_2 \cos\omega_2 \cdot t, \quad (20)$$

where $\omega_1 = \omega_o \sqrt{(1+\mu)}$; $\omega_2 = \omega_o \sqrt{(1-\mu)}$.

The constant values C_1, C_2, D_1, D_2 in the equations (19) and (20) are determined from the conditions "of reproduction" through the period T and the continuity of solution with $t = \frac{T}{2}$

$$x_1(T/2) = x_2(T/2); \quad \dot{x}_1(T/2) = \dot{x}_2(T/2).$$

Boundaries of the region of instability, which determine the curve in the plane of parameters of modulation depth μ and the relationships of the periods $\alpha = \omega_0 \frac{T}{2\pi}$ (relations between the average value ω_0 of natural frequency and the frequency of parametric action v) are calculated from the relationship

$$\cos\left(\frac{\omega_1 T}{2}\right)\cos\left(\frac{\omega_2 T}{2}\right) - \quad (21)$$

$$-\frac{\omega_1^2 + \omega_2^2}{2\omega_1\omega_2} \sin\left(\frac{\omega_1 T}{2}\right)\sin\left(\frac{\omega_2 T}{2}\right) = 1$$

The motion of a technological system with harmonic local action can be described in the form of the differential equation of Mathieu

$$\ddot{x} + \omega_o^2 (1 + 2\mu\cos\nu t) x = 0, \quad (22)$$

where ω_0 - the average value of natural frequency; μ - the depth of modulation.

The solutions of this equation are special functions, called Mathieu functions, with the known properties [5]. For determining the boundaries between the region of stability and the region of instability on the plane of parameters *a* and *ε*, it is convenient to use a method of harmonic balance.

Unstable regions of the Mathieu-Hill equation on the plane *μ*, *ν* adjoin the frequencies of the relationships $\nu = \frac{2\omega_0}{p}$ $(p = 1,2,...)$. The relative width of the region of main parametric resonance is of the order *μ*. With the sufficiently small values of *μ* the boundaries of this region are calculated from the formula

$$\nu = 2\omega_o \sqrt{(1 \pm \mu)}. \quad (23)$$

The presence of damping of a certain level makes the parametric excitation impossible if *μ* is sufficiently low. If this is the case, the higher the order *p* of the side-line parametric resonance, the stronger is the effect of damping.

Since the periodic alternation in the parameters of the system in question, caused by local action, occurs with the frequency, which is considerably lower (50…100 times) than that of the natural vibrations of the system [2], it is possible to make a conclusion about the stability of the system by changing the frequencies of local action and these of natural vibrations within the range of technologically defined limits.

Considering local action in the system with increasing fluctuations as the action of asynchronous forced oscillations, an equation, in its general form, can be represented

$$\ddot{x} + 2h\dot{x} + \omega_0^2 x = F + P_{lpa}\cos(\nu t), \quad (24)$$

where $F = \frac{F(x,\dot{x})}{m_x}$ - the value, determined by the variable component of all forces of the working process, which takes place in the cutting zone; $P_{lpa}\cos(\nu t)$ - the value, determined by the force formed as a result of the local action; ν - the frequency of the local actions. Approximating the variable component [4, 6], we obtain

$F = -S_1 x - S_2 x^2 + S_3 x^3 + (-S_1 - 2S_2 x + 3S_3 x^2)x,$

where $S_i (i = 1,2,3)$ - the coefficients, the values of which depend on the parameters of the process of cutting and physical-mechanical properties of the workable material.

Assuming that the increasing fluctuations, which appear in the process of mechanical processing, are almost harmonic, equation (24) can be represented in the form

$$\ddot{x} + \omega_0^2 x = \left(\omega^2 - \omega_0^2\right)x + 2h\dot{x} + F + P\cos(\nu t). \quad (25)$$

Like with the asynchronous effect of extra forces, which appear in the zone of LPA, on a system with increasing fluctuations, all regions of forced synchronization are excluded from examination, in the present case two types of fluctuations are established in the system: the parametric fluctuations of frequency ν and the increasing fluctuations of frequency ω that are close to the natural frequency of the prevailing system ($\omega \approx \omega_0$), and are not connected with the frequency of parametric action. Therefore, the resulting motion can be represented as the sum of harmonic fluctuations

$$x = a\sin\omega t + \lambda\cos\nu t. \quad (26)$$

Under the asynchronous influence, for the conditions, whereby $\omega >> \nu$, the solution for “forced” fluctuations can be found from the examination of the conservative part of the equation (24)

$$\ddot{x}_1 + \omega_0^2 x_1 = P\cos(\nu t);$$

$$x_1 = \frac{P}{\omega_0^2 - \nu^2}\cos\nu t; \quad \lambda = \frac{P}{\omega_0^2 - \nu^2}. \quad (27)$$

Considering the right side of equation (26) as the sum, resulting from the forces, which act on the conservative oscillatory system, and converting it on the basis of equation (26), we obtain

$$\Sigma F_k = \begin{pmatrix} \omega^2 - \omega_0^2 + S_1 - S_3 a^2 \sin\omega t - \\ 3S_3\lambda^2\cos^2\nu t \end{pmatrix} \times$$
$$\times a\sin\omega t + \begin{pmatrix} -2h + S_1 - 3S_3 a^2\sin^2\omega t - \\ 3S_3\lambda^2\cos^2\nu t \end{pmatrix} \times \quad (28)$$
$$\times u\,\omega\cos\omega t + \ldots$$

In the conservative oscillatory system, described by the left side of equation (25), the purely harmonic oscillations of resonant frequency can exist only in the absence of LPA. Consequently, during the steady self-oscillatory motion the forces present on the right side of equation (25) must become zero. On the basis of this, from equation (28) we compose a system of equations for determining the steady values of amplitude and frequency of the self-oscillatory motion of the technological system

$$\begin{pmatrix} \omega^2 - \omega_0^2 + S_1 - S_3 a^2 \sin\omega t - \\ 3S_3\lambda^2\cos^2\nu t \end{pmatrix} \cdot a\,\omega = 0; \quad (29)$$
$$\begin{pmatrix} -2h + S_1 - S_3 a^2 \sin^2\omega t - \\ 3S_3\lambda^2\cos^2\nu t \end{pmatrix} \cdot a\,\omega = 0.$$

After conversions we obtain an equation for determining the amplitude of auto-oscillations

$$\left(-2h + S_1 - \frac{3}{4}S_3 a^2 - \frac{3}{2}S_3\lambda^2\right)a\,\omega = 0, \quad (30)$$

from which we determine the value of the steady amplitude in the absence of parametric action, i.e. with $\lambda = 0$

$$a_0 = \pm\sqrt{\frac{4(S_1 - 2h)}{3S_3}}. \quad (31)$$

Taking into account (25), (26) and (27) we obtain the value of the steady amplitude

$$a_0 = \pm\sqrt{\frac{4(S_1 m_x - b_x - \beta_3)}{3S_3 m_x}}. \quad (32)$$

In the presence of LPA the amplitude of auto-oscillations in the technological system will be determined by the following expression

$$a = \pm\sqrt{\frac{4(S_1 m_x - b_x - \beta_3)}{3S_3 m_x} - 2\lambda^2}. \quad (33)$$

In the absence of auto-oscillations in the technological system, in the course of mechanical processing $a = 0$, therefore, the extinction condition, defined for auto-oscillations under the asynchronous parametric influence of the extra force, which appears in the zone LPA, is given as

$$\lambda_{abs} \geq \sqrt{\frac{2(S_1 m_x - b_x - \beta_3)}{3S_3 m_x}}. \quad (34)$$

Following from (27) the amplitude of force, caused by the parametric action, must satisfy the condition

$$P'_{lpa} \geq (\omega_0^2 - \nu^2)\, m_x \sqrt{\frac{2\,(S_1 m_x - b_x - \beta_3)}{3\,S_3 m_x}} \qquad (35)$$

and with regard to the value a_0 (32), it is possible to represent it in the following form

$$P'_{lpa} \geq \frac{a_0(\omega_0^2 - \nu^2)\, m_x}{\sqrt{2}}. \qquad (36)$$

After the substitution in (35) of the value $\omega_0^2 = \frac{c_x + c_3}{m_x}$ expression for the necessary amplitude of a change in the force of the parametric action takes the form

$$P'_{lpa} \geq \left(\frac{c_x + c_3}{m_x} - \nu^2\right) \cdot \sqrt{\frac{2\,m_x(S_1 m_x - b_x - \beta_3)}{3\,S_3}}. \qquad (37)$$

Thus, when the local effect in the process of cutting on the prevailing system located beyond the region of stability has the amplitude, which is determined by dependencies (35) - (37), the suppression of the appearing auto-oscillations occurs.

4. CONCLUSIONS

1. We have hereby developed the method, which is based on the local action on the surface of material, leading to a change in the crystal lattice of the local zone, the generation of high-energy configurations and the appearance of increased metastability of structure in this local region. In comparison with the source material it allows to ensure a periodic alternation of processing conditions in the local zone in the process of cutting.
2. Theoretical studies of the periodically variable parameters of the system in question, caused by local action, are accomplished with the frequency, which is considerably lower (50…100 times) than that of the natural vibration of the system, which makes it possible to ensure the stability of the system by a change in the frequencies of the local action and those of natural vibrations within the range of technologically defined limits.
3. The low-frequency parametric local physical effect on potentially self-oscillatory system with the vibration frequency, which considerably exceeds the frequency of action LPA can be interpreted as the imposition on the system additional forced fluctuations of frequency equal to the frequency of local action, and of amplitude equal to the difference in the amount of the static cutting forces, resulting from the difference in the mechanical properties of metal in the zones studied.

5. REFERENCES

1. Eljasberg M.E. Auto-oscillations of the machine tools. Theory and the practice. – Sankt-Petersburg, Ed: OKBS, 1993. – 180 p. (in Russian).
2. Veitz V.L., Maksarov V.V. Increased stability of a technological system in the management of rheological parameters of the process chip formation//Mechanical Engineering and Automation: Ed: Academia collection. Vol. 16. – Sankt-Petersburg, NWTI, 1999. – pp.19 – 29 (in Russian).
3. Maksarov V. V., Olt J. Managing the chip by the prior local plastic effects on the work surface workpiece// Academia News, Mechanical Engineering, 2008, Vol 6. - pp. 45-51 (in Russian)
4. Panovko J. G. The internel friction during the vibrations of elastic systems. – Moskow, Fizmatgiz, 1960. – 193 p. (in Russian).
5. Podurajev V.N., Zakurajev V.V. Develop and implement ways to control the optimum cutting // Journal of Mechanical Engineering.– 1996. – Vol 11. – pp. 31-36 (in Russian).

6. CORRESPONDING ADDRESS

Jüri Olt, Institute of Technology, Estonian University of Life Sciences, Kreutzwaldi 56, 51014 Tartu, Estonia, E-mail: jyri.olt@emu.ee.