A STOCHASTIC PROGRAMMING MODEL FOR SUPPLY CHAIN PLANNING

Küttner, R.

Abstract: The paper proposes both deterministic and stochastic programming models for manufacturing planning for supply chain network. A two-stage scheme for stochastic programming models is proposed. The design objective is to maximize the total profit. For modeling we deal with uncertainty by making randomness in the data explicit. In general, we may have randomness in all of model data, such as “processing times”, “the actual demands” etc. We propose that we have scenario trees, where each scenario has some probability of occurrence. A computational study involving different supply chain networks are presented to highlight the significance of the stochastic model.

Key words: supply chain planning, deterministic and stochastic programming models, two-stage model.

1. OPTIMAL PLANNING OF THE SUPPLY CHAIN (DETERMINISTIC MODEL)

2.

In the 90’s system theory has strongly influenced process management. Instead of examining single processes or enterprise, nowadays networks of interacting processes or enterprises (supply chains) are analyzed. A supply chain structure can be viewed as a network of suppliers, manufacturing plants, transporters, warehouses, and customers organized to acquire raw materials, convert these raw materials to finished products, and distribute these products to customers. The deterministic model is based on the planned requirement for products or product components, described by the product structure (Bill of Material).

In a highly competitive environment, a supply chain should be managed in the most efficient way, with the objective of:

- minimization of cost, delivery delays, inventories, and investment;
- maximization of profit, return of investment (ROI), customer service level, etc.

The planning tasks could consist different decisions, with time horizons ranging from several years to a few days, respectively:

1. Location decisions consider the number, size and physical location of production plants, warehouses, and distribution centers;
2. Production decisions consider the products to be produced and production schedules at each plant;
3. Inventory decisions are concerned with the management of the inventory levels;
4. Transportation decisions include the transportation media to be used for and size of each shipment of material and components.

Strategic level supply chain planning involves the configuration of the network, i.e. number, capacity, and technology of the facilities. The tactical level planning of supply chain operations involves deciding the aggregate quantities and flows for purchasing, processing and distribution of products [1].

Let us first describe a deterministic mathematical formulation for the supply chain planning problem. Consider as an example a supply chain network...
\[ \Theta = (N, A) \] (Fig 1), where \( N \) is the set of nodes and \( A \) is the set of arcs.

Figure 1. Schema of simple supply chain network \( \Theta = (N, A) \), where nodes \( N \) are representing: \( S_i \) - suppliers; \( E_j \) - processing facilities; \( C_u \) – customers, and arcs are representing the material flows for different products.

The set \( N \) consists of the set of suppliers \( S \), the set of manufacturing enterprises \( E \) and the set of customers \( C \), i.e., \( N = S \cup E \cup C \). The manufacturing enterprises \( E_i; i = 1, n \) include different workstations (manufacturing centers) \( M \), and assembling facilities \( F \), i.e., \( E = M \cup F \). Components are purchased by different suppliers \( S_j ; j = 1, m \) and suppose that there are different customers \( C_u; u = l \).

Let \( P \) be the set of products flowing through the supply chain. We have \( k \) products (\( P_1, P_2, ..., P_k \)), which are assembled out of \( m \) components (\( x_1, x_2, x_3, ..., x_m \)). The structures of products are flat and could be represented in form of Gozinto Graph. A Gozinto Graph is a tree-like representation of raw materials, parts, intermediates and subassemblies, in which a particular production process transforms into an end product through a sequence of (production) operations. The representation of the Gozinto graph by data structure is referred to as the Bill of Material (BOM) (see Table 1). The BOM registers each relation between a subordinate component and super-ordinate component and compiles the list of components required for the production of parent items.

<table>
<thead>
<tr>
<th>( \text{x1} )</th>
<th>( \text{x2} )</th>
<th>( \text{x3} )</th>
<th>...</th>
<th>( \text{xm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>...</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>( ... )</td>
<td>( ... )</td>
<td>( ... )</td>
<td>( ... )</td>
<td>( ... )</td>
</tr>
<tr>
<td>( P_n )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>...</td>
</tr>
</tbody>
</table>

To solve the task we need a list of processing/purchasing times \( a_{i,j} \) of each component on each enterprise and machining centre.

The supply chain configuration decisions consist of deciding which of the processing centers to build and which machines to procure. We associate a binary variable \( y_i \) to these decisions, \( y_i = 1 \), if a processing facility \( i \) is built or machine \( i \) is procured, and 0 otherwise. The operational decisions consist of routing the flow of product
$p^k \in P$ from the supplier to the customers. We let $x^k_{ij}$ denote the flow of product $p^k$ from a node $i$ to a node $j$ of the network where $(ij) \in A$.

The planning task consists both tasks, configuration decisions and operational decisions with the objective to maximize the total profit as sum of selling price minus current investment costs and expected processing and transportation costs.

Max

$$\Pi = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( r_i \times X^k_{ij} - y_w \times \text{Inv}_w - X_{jj} \times (M_i + c_j^k \times a_{i,j} \right)$$

Subject to:

1. $\sum_{i=1}^{m} X^k_{ij} - \sum_{i=1}^{m} X^k_{ji} = 0; \forall j \in E; \forall k \in K$

balance of material flow for each products and nodes;

2. $d_{i,\delta}^{\min} \leq X_{i,\delta} \leq d_{i,\delta}^{\max}$ for all product variants $i$ and customers $\delta$;

3. $\sum_{i=1}^{m} a_{i,w} \times X_{i} \leq C_{w} (1 + y_{w})$ processing times for all manufacturing centers (enterprises) $w$;

4. $\sum_{i=1}^{m} X_{i,u} \times M_{i} \leq \mu_{i}$ for all materials and components (suppliers) $u$;

5. $X_{i,j} \geq 0$, for all $i,j$.

6. $y_{w} \subseteq \{0,1\}$

where
- $\Pi$ - objective function, total profit
- $r_i$ - net profit from one unit of sold product $p^i$;
- $a_{i,j}$ - time required for processing product $i$ on machine (enterprise) $j$.
- $c_j^k$ - per-unit cost of processing product $k$ at facility $j$ (or transporting product $k$ on arc $(ij)$)
- $C_w$ - capacity of processing unit $w$
- $M_i, \mu_i$ - cost and resource of material $u$;
- $\text{Inv}_i$ - investment cost for building facility $i$ or procuring additional machine $i$ required for implementing product $p^i$;
- $X_i$ - quantity of products $p_i$ produced during the period analyzed;
- $y_{i}$ - indicator of processing facility $i$ is build or not.

1. THE STOCHASTIC MODEL

Uncertainties that occur throughout the entire supply chain network, are affecting its performance and generally its operation efficiency. To extend the above model to a stochastic setting, we consider uncertainty arising from suppliers, manufacturing, and customers. Suppliers can be characterized through their past performance, and their responsiveness can be predicted. For manufacturing we suppose that there are possibilities to change the characteristics of the technological processes and organization of processes. Finally, a customer demands involve uncertainty which needs to be addressed via forecasting methods.

A way to deal with the stochastic nature of parameter $\xi$ is to use the stochastic scenario tree (Figure 2), where the right nodes of tree represent different states of parameter $p_i$ (or scenarios). Each scenario has some probability $\omega_i$ of occurrence, which can be an objective measure derived by statistical information or a subjective measure of likelihood. Scenarios can be the result of a discretization of a continuous probability distribution $f_p(\xi)[i]$. 
We use bold face to denote random variables (random vectors) in order to distinguish them from their particular realizations. In particular, \( \hat{\xi} = (c_j, a_{i,w}, r_i, \mu_\omega) \) represents the random data vector while \( \xi = (c_j, a_{i,w}, r_i, \mu_\omega) \) stands for its particular realization. The resulting formulation (for stochastic demand as example) is as follows:

**Max**

\[
\Pi = \sum_{i,m} \sum_{j=1}^{i-k} (r_i \times X_{ij}^k - y_{w} \times Inv_{w} - X_{ij} \times (M_i + c_j \times a_{i,j})
\]

Subject to:

1. \( \sum_{i=1}^{N} X_{ij}^k - \sum_{i=1}^{K} X_{ij}^k = 0; \forall j \in E; \forall k \in K \)

balance of material flow for each products and nodes; 2. \( X_{ij}^k \leq \hat{d}_{k,\delta} \)

random demand for all product variants \( k \) and customers \( \delta \);

3. \( \sum_{i=1}^{m} a_{i,w} \times X_{ij} \leq C_u (1 + y_{w}) \) for all manufacturing centers (enterprises) \( w \);

4. \( \sum_{i=1}^{m} X_{ij,\mu} \times M_i \leq \mu_i \) for all materials and components (suppliers) \( u \);

5. \( X_{ij} \geq 0, \) for all \( i,j \).

6. \( y_{w} \subseteq \{0,1\} \)

where

- \( \hat{d}_{k,\delta} \)-stochastic demand for product \( k \) and customer \( \delta \).

In Table 2 the results of simulation for stochastic demand are represented.

Table 2. Example of variation of solutions considering stochastic demands, where \( X_{ij} \) are the volumes of products produced and \( Y_{ij} \) number of additional machines procured.

<table>
<thead>
<tr>
<th>Profit</th>
<th>X_1</th>
<th>X_2</th>
<th>X_3</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
<th>Y4</th>
<th>Y5</th>
</tr>
</thead>
<tbody>
<tr>
<td>25104</td>
<td>100</td>
<td>160</td>
<td>150</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>24840</td>
<td>120</td>
<td>120</td>
<td>150</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>26216</td>
<td>120</td>
<td>160</td>
<td>150</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>28256</td>
<td>120</td>
<td>160</td>
<td>175</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>27080</td>
<td>120</td>
<td>120</td>
<td>175</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>28256</td>
<td>120</td>
<td>160</td>
<td>175</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>28256</td>
<td>120</td>
<td>160</td>
<td>175</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>29320</td>
<td>120</td>
<td>120</td>
<td>200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The proposed stochastic model could be represented as a two-staged stochastic program \([1, 2]\). The first stage consists of the deciding the configuration decision \(y\), and the second-stage consists of processing (and transportation) products from suppliers to customers in an optimal fashion based upon the configuration and the realized uncertain scenario. The objective for the first stage is to minimize the expected investment costs \(E[Q(y, \xi)]\):

\[
\min_y \quad f(y) := E[Q(y, \xi)]
\]

\[
y \in Y \subseteq [0, 1]^p
\]

\[
\sum a_{i,w} X_i \leq W_u (1 + y_u)
\]

The optimal value \(Q(y, \xi)\) of the second stage problem is a function of the first stage decision variable \(y\) and a realization (or a scenario) \(\hat{\xi} = (c_j, a_{i,w}, r_i, \mu_u)\) of the uncertain parameters. \(E[Q(y, \xi)]\) is estimated as “response surface” model solving the second stage problem and using for example the regression analysis. The expectation \(E[Q(y, \xi)]\) is taken with respect to the probability distribution of \(\hat{\xi} = (c_j, a_{i,w}, r_i, \mu_u)\).

For our simple model we have

\[
E[Q(y, \xi)] = 26800 + 0^*y_1 - 477.3^*y_2 + 0^*y_3 + 0^*y_4 + 2659.6^*y_5
\]

There are two potential sources of difficulty in solving proposed problem:

1. An evaluation of the objective function \(f(y)\) (for a given configuration \(y\)) involves computing the expected value of the linear programming value function \(Q(y, \xi)\). For discrete distribution, computing the expectation might involve solving a large number of linear programs of the second-stage problem, one for each scenario of the uncertain problem parameter realization.

2. Even if the expectation \(E[Q(y, \xi)]\) can be computed exactly, optimization of this function presents significant difficulties, \(E[Q(y, \xi)]\) is a convex non-linear function of \(y\). Thus problem involves maximizing an implicitly defined non-linear function with respect to binary variables, and is quite difficult. The function \(E[Q(y, \xi)]\) could be approximated for example using piecewise linear function \([2]\).

We deal with the problem using the SAA (Sample Average Approximation) scheme. \([1]\).

In the SAA scheme, a random sample \(\xi_1, \ldots, \xi_N\) of \(N\) realizations (scenarios) of the random vector \(\xi\) is generated, and the expectation \(E[Q(y, \xi)]\) is approximated by the sample average regression function. For a particular realization \(\xi_1, \ldots, \xi_N\) of the random sample, the problem is deterministic and can be solved by appropriate optimization techniques.

\(Q(y, \xi)\) is the optimal value of the following second-stage problem:

**Max**

\[
\Pi = \sum_{i=1}^{i=m} \sum_{j=1}^{j=k} (r_j \times X_{i,j} - X_{i,j} \times (M_j + c_j \times a_{i,j})
\]

**Subject to:**

1. \(\sum_{i=N} X^k_{ij} - \sum_{i=N} X^k_{ji} = 0; \forall j \in E; \forall k \in K\)

   balance of material flow for each products and nodes;

2. \(X^k_\delta \leq d_{k,\delta}\) for all product variants \(i\) and customers \(\delta\);
3. \[ \sum_{i=1}^{n} a_{iw} \cdot X_i \leq C_w(1 + y_w) \] for all manufacturing centers (enterprises) \( w \);

4. \[ \sum_{i=1}^{n} X_{i,u} \cdot M_i \leq \mu_i \] for all materials (suppliers) \( u \);

5. \( X_i \geq 0 \), for all \( i \)

Based on the proposed models, planning tasks are represented as a mixed integer and combinatorial 0-1 programming problems. The possibilities of Excel Solver were used to solve different examples.

3. CONCLUDING REMARKS

In the paper we have developed a methodology for planning the manufacturing of products for supply chain. This paper proposes both deterministic and stochastic programming models for solving manufacturing planning problems for supply chain network. Supply chain planners face a significant amount of uncertainty, particularly during the strategic planning phase. For stochastic set up we have considered as an example a demand uncertainty. The proposed stochastic methodology is based on statistical simulation and on use of sample average approximation scheme. The proposed approach is acceptable in case of higher variability and multiple resources of uncertainty.

We hope that this paper will help to develop future research on supply chain planning topics.

4. REFERENCES


