

OPTIMAL PLANNING OF PRODUCT MIX FOR SUBCONTRACTING COMPANIES

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Abstract: *Aggregate planning models as linear program (LP) are formulated in this paper.*

The uses of models for various examples of production outsourcing and planning the optimal Product Mix are demonstrated. The results of analyzes for different situations are given.

The models to solve the problem of optimal Product Mix development, conditions determination under which outsourcing is profitable for a firm are proposed.

The goal is not so much to provide specific solutions to particular problems, but rather to illustrate general problem-solving approaches.

Keywords: *Aggregate planning. Production outsourcing. Managing the Product Mix. Decomposition of aggregate planning task.*

1. INTRODUCTION

Competitive manufacturing involves careful study of potential markets, customer's needs, design of products, understanding of the processes of manufacturing, organization and planning of production, and rational use of resources.

The paper is concerned with coordinated design of products and manufacturing processes, planning of production so that effective and efficient manufacturing will be possible, to combine the problems of production development and production planning into one integrated model.

The product and process requirements indicate what technologies are feasible or what product and their manufacturing process characteristics must be, what product should be developed. An economic choice is to be made among technically feasible alternatives. The resources required to complete all manufacturing operations must be minimized; the gained profit must be maximized. Sophisticated economical analysis, models, and computer tools must support the decision-making process.

The main examples of engineering decisions include:

- **Managing the product mix:** to make decisions about Product Mix and production volumes in production, which products to promote on the basis of demand forecast and knowledge of which products have tight capacity restrictions and which do not.
- **Production outsourcing:** to make decisions, which components to make in-house and which contract out to external suppliers to manufacture components or to perform specific operations.

The problem is named as **aggregate planning** (AP) [1]. The AP integrates different interrelated aspects of production into one model.

Based on use of AP the systematic decision-making involves all aspects of the product and production planning in strategic point of view. The AP uses the marketing information and generates forecasts of future demands together with description of manufacturing process requirements for making the various products in Product Mix, to determine needs for equipment and other resources. AP includes also typically the

solutions of inventory control, capacity planning and other problems.

2. BASIC MODEL

The model calculates explicitly the Product Mix that maximizes overall profit considering available resources and demand for multiple products. Resources are anything that is needed to produce or provide the family of products, and may include materials, labour, machine time, floor space, transportation services etc.

Because the different products require different amounts of processing time on the various workstations, the workstation that is most heavily loaded during a period may be clearly dependent on the mix of products, which run during that period.

We introduce the following notations:

i = An index of product, $i=1, \dots, m$, so m represent the total number of products

j = An index of workstation, $j=1, \dots, n$, so n represent to total number of workstations

t = An index of time period, where $t=1, 2, \dots, tl$, where tl is the planning horizon for the problem

d_{it}^{\max} = Maximum demand for product i in period t

d_{it}^{\min} = Minimum sales allowed of product i in period t

a_{ij} = Time required on workstation j to produce one unit of product i

C_{jt} = Capacity of workstation j in period t in units consistent with those used to define a_{ij}

S_i = Selling price of product i .

r_i = Net profit from one unit of product i

h_i = Cost to hold one unit of product i for one period t ; for example, if holding cost consists entirely of interest on money tied up in inventory, then $h_i = i * C_i / 52$, where i is the annual interest rate and periods correspond to weeks. The holding cost for one period and associated with an average inventory is determined as $h * (I_{it} + X_{it} / 2)$.

C_i = Unit production cost (not accounting inventory costs) product i .

X_{it} = Quantity of product i produced during period t (assumed available to satisfy demand at end of period t)

S_{it} = Quantity of product i sold during period t (we assume that units produced in t are available for sale in t and thereafter)

I_{it} = Inventory of product i at the end of period t (after demand has been met); we assume I_{i0} is given as data.

The task is to find a function $F(X, S, I, D, C, I_0)$ that maximizes net profit minus inventory-carrying and holding costs subject to upper and lower bounds on sales and capacity constraints, where X, S, I are the vectors of decision variables and D, C, I_0 represent the vectors of input data: demand, capacity and inventory at the begin of the planning period.

We can give a following linear program formulation of the task $F(X, S, I, D, C, I_0)$:

$$\text{Max} \sum_{t=1}^{it} \sum_{i=1}^m s_i * S_{it} - C_i * X_{it} - h_i * (I_{it} + X_{it} / 2)$$

sales revenue-inventory and holding cost

subject to:

$$d_{it}^{\min} \leq S_{it} \leq d_{it}^{\max} \quad \text{for all } i, t \text{ demand}$$

$$\sum_{i=1}^m a_{ij} * X_{it} \leq c_{jt} \quad \text{for all } j, t \text{ capacity}$$

$$I_{it} = I_{it-1} + X_{it} - S_{it} \quad \text{for all } i, t \text{ inventory balance}$$

$$I_{it} \geq s_{it} \quad \text{for all } i=1, m, t=0 \text{ requirements for safety stock}$$

$$X_{it}, S_{it}, I_{it} \geq 0 \quad \text{for all } i, t. \text{ non-negativity}$$

Basic formulation contains capacity constraints for the workstations, but in some situations also other resources, such as people, raw materials, transport device capacity, allowed maximum for inventory (capacity of store(ware)houses), etc may be important determinants [1].

Generically, if we let

$$b_{ij} = \text{units of resource } j \text{ required per unit of product } i$$

$$k_{jt} = \text{number of units of resource } j \text{ available in period } t,$$

we can express the general capacity constraints on resource j in period t as

$$\sum_{i=1}^m b_{ij} * X_{it} \leq k_{jt}$$

The other considerations [1], which could be considered in AP model are:

- Backorders
- Overtime
- Yield Loss
- Workforce Plan etc.

The planning time is divided into buckets. In examples, the bucket length is considered as a week. In numerical examples, a situation, where a firm on three workstations produces two final products, each of which consist 5 to 6 different items, is considered.

There is 2400 minutes time available on workstations. For each product selling prices, material, components costs and required manufacturing times on workstations are given as input variables.

Figure 1 illustrates the impact of market demand D (volume of production) to the total profit $F(X, S, I, D, C, I_0)$. Three different regions of demand could be recognized: region A – the production volumes and profit are restricted only by the market demand, the available resources (capacity of workstation) are not restricting the optimal decisions; region B – the increase of demand give some increase of profit, the given resources are restricting the possibilities of a firm to satisfy the

market demand; region C - the firm is not able to satisfy the market demand due to restricted production capacities.

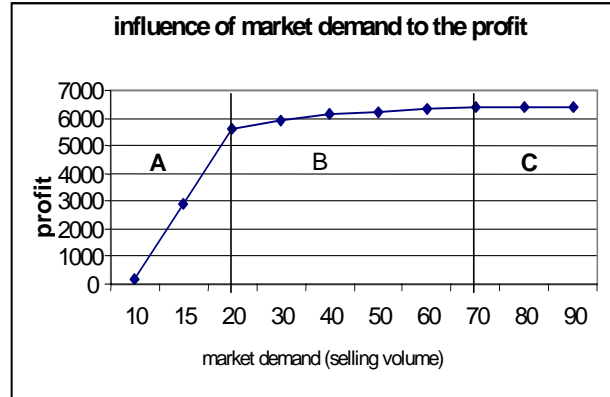


Fig 1. Change in profit due to increase of market demand (selling volume)

3. THE OPTIMAL OUTSOURCING OF PRODUCTION

The management of a company seeks to operate within the chain of cooperating enterprises. Outsourcing of production occurs when a firm (the original producer, OP), instead of completing the entire production in-house, employs an outside contractor (OC) to make a part of the product or the process outside by utilizing OC's resources [3]. Because OC could be a specialized or bulk producer, it may be more efficient if OC is involved in the design and production of the components. A firm can outsource the production of some product or component completely, partially, or not at all. If an operation or product (component) is a core or strategic one, it is kept in-house so that control is retained. If a component is a standard component, commodity or if suppliers can provide it for a less costs, it may be outsourced when additional transportation and managerial costs are not substantial. The model based on AP is needed to determine the decision of what fraction of components to produce in-house and what fraction to outsource.

The OP's profit from own production (before outsourcing) is calculated by function $F(X, S, I, D, C, I_0)$.

The optimal outsourcing is determined by dividing the Product Mix (family of products) X^o into two parts $X^o = (X_1, X_2)$, where X_1 is produced in-house and the production of component X_2 is outsourced to OC.

The profitable outsourcing requires that $F(X^o, S, I, D, C, I_0) \leq F(X, S, I, D, C, I_0)$, and requires that for the OP the savings in unit production cost, inventory and holding costs exceed the cost of buying and transportation of X_2 from OC.

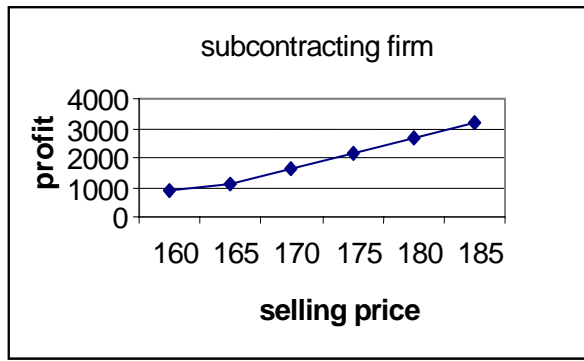


Fig2. Effect of increasing of selling price on profit of contracting company OC (OC is a specialized (bulk) producer).

Figures 2 and 3 give the results, as examples, of profit changes simulations for OC and OP due to changes in selling/purchasing price of a component.

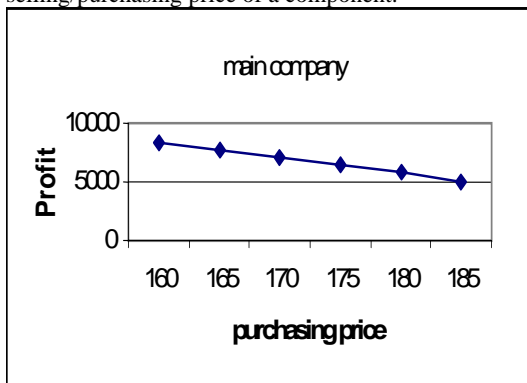


Fig 3. Effect of decreasing of purchasing price on the profit of main company OP.

The degree of outsourcing differs widely across firms. Different situations to determine the optimal degree of outsourcing could be estimated numerically using AP. The results of analyzes form a good basis for negotiations between OP and OC.

4. THE PLANNING OF OPTIMAL PRODUCT MIX

Product Mix planning models are well-known class of problems that have been investigated since the end of 1950's. In the classic Product Mix planning model, a decision maker selects the volume and mix of product that maximizes profit, subject to constraints on demand and production resources.

If the flexibility in the mix is possible, we can use the AP module to adjust the mix in accordance with available capacity in company. For the fixed product mix we can use the AP model to obtain different information, including:

1. **Demand feasibility estimations:** to determine whether a set of demands for product mix for given period is capacity-feasible.
2. **Bottleneck recognition:** by noting which constraints limit capacity in given period.
3. **Product Mix adjustment:** if it is for capacity reasons unable to attain the upper bounds on demand, then manager can reduce the volumes of some products in Product Mix and try to maximize revenue by producing products with higher profitability.

The models of AP could be used to determine appropriate production quantities of all types of items in Product Mix, from final products, and to lower-level items (components). Demand for final products is independent demand. Typically most

demand for lower-level items is dependent demand. However there can be independent demand for lower-level items if these items are, for example, end items for subcontracting companies. The relationship between end items and lower level items is described by the Bill of Material (BOM), as a list of all the subassemblies, parts, raw materials and purchased components that go into final product [2]. The AP gives a good model to determine the optimal Product Mix, to analyse numerically the influence of widening the list of products in Product Mix. In figure 4 an example of changes of total profit as function of share of product X in Product Mix.

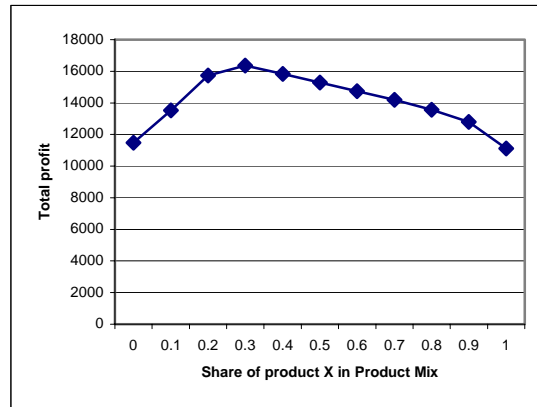


Fig 4 Product Mix adjustment (example with different shares of product X)

5. DECOMPOSING THE BASIC MODEL

The central theme in AP is the overwhelming complexity of the planning task. The simplification of AP task is based on decomposition of initial planning task. Using decomposition each of these sub-tasks could be treated separately.

The essential ways to decompose the AP model to sub-models are:

- The optimal outsourcing of production by decomposing the initial problem between the various subcontracting enterprises
- Decomposing the initial Product Mix problem between different products.

There are several reasons for decomposition of the global task:

- The division initial task into a smaller sub-tasks lead to better understanding of the whole problem.
- Breaking down the functional subsystems may lead to the realisation of AP in different functional units/enterprises; there may be some already existing subsystems/models that can provide useful information.
- The resulting decomposition controls the search for solution of initial problem.

Usage of different decomposition approaches raise question of equivalence between initial AP task and the tasks represented by the decomposition schemes.

Main ways to implement the decomposition and related to decomposition co-ordination are [3]:

- to add to subtasks, so called, "auxiliary variables" representing the initial "guesses" or prognoses (forecast) of the mutual influences for tasks, typical and recommendable solutions (market demand, purchasing prices etc), parameters (for example production volumes) etc;
- to add additional constraints that represent the convergence restrictions on possible solutions and the set of feasible solutions for subtasks. For example, sharing the resources, used to produce different products in Product Mix, restriction of resources for transport (transport devices) etc;

- to tune the objective function of the subtasks, giving some priorities for subcontracting companies for implementation of a new product etc.

For example, to decompose the Product Mix for different products and/or components a fraction q of shared production resource capacity j could be used as additional constraints for co-ordination:

$$\sum_{i=1}^m a_{ij} * X_{it} \leq q * c_{jt} \text{ for all } j,t$$

A two-stage procedure proposed in [4] could be used to generate the optimal solution of adding a new product to the existing product mix and to estimate the optimal conditions for outsourcing the production of some components of a product.

A basic model of AP is first applied to generate an initial solution. In the second stage the initial product mix is evaluated using shared resource approach. Changing the parameters of the bottleneck of workstation group modifies the product portfolio.

The improvement process of planning continues for a number of iterations until all requirements are met in an optimal way.

In the second stage, there could be multiple views of what exactly the bottlenecks are. Different bottleneck indicators have been compared for effectiveness. They are utilizations, queuing delay and remnant capacity.

A series of product mixes can be generated and estimated by using each indicator.

6. CONCLUDING REMARKS

The paper is concerned with coordinated design of products and manufacturing processes, planning of production. Sophisticated models for analysis and computer tools are proposed to support the decision-making process. The proposed decision model, based on linear planning approach for aggregate planning, could be applied to estimate objectively arbitrate capacity sharing between different co-operating firms and to evaluate the economic value of shared resources, to plan the optimal Product Mix for a company.

The objective of the paper is not so much to provide specific solutions to particular problems in companies, but rather to illustrate general problem-solving approaches.

Finally, the principles of initial AP task decomposition are presented.

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