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VIRTUAL DIFFERENTIAL AS TORQUE DISTRIBUTION CONTROL UNIT IN AUTOMOTIVE PROPULSION SYSTEMS

Resev, J. & Roosimölder, L.

Abstract: *The energy flow modelling in automotive propulsion systems aimed to provide base data for the choice of the differential parameters in consideration of the impact on the lateral dynamics of the vehicle is presented in the paper.*
The characteristics of the differential in the torque transmission chain are concerned. Description of the characteristics and operational conditions of the limited slip differential gear train is given. The virtual differential (algorithm of energy flow) has been provided.
The topics of this paper have been developed in co-operation between Chalmers University of Technology, Göteborg, Sweden, Tallinn Technical University and Estonian Agricultural University.
Key words: modelling, planetary gear train, limited slip, torque transmission, energy losses.

1. INTRODUCTION

Development of the control unit for the vehicle (its neural system) to improve the safety characteristics needs to control the power flow from the engine to the driving wheels.
This paper concerns the characteristics of the differential in the torque transmission chain.
An essential disadvantage of the conventional differential has been noted: in case one wheel of the vehicle slips on a surface with low friction, it is likely to bring the vehicle to halt. In the above case, the conventional differential is unable to transmit the necessary torque to the other wheel. The limited slip differential can transmit more torque in this case but a decrease in the steering qualities will follow due to the increased understeering.
The aim of this work is to investigate the power flow from the engine to the driving wheels in consideration of its impact on the lateral dynamics of the vehicle.

2. NOTATION AND TERMS

main symbol	**quantity, explanation**	**unit**
A	area	m^2
B	track	m
F	(peripheral) force	N
G	gravitational force	N
g	acceleration of gravity	m/s^2
L	wheel base	m
J, $[J]$	mass moment(s) of inertia	kgm^2
$[J_C]$	constraint Jacobian matrix	[m]
P	pitch point	W
r, R	radius	m
Δr	force pole offset	m
s	slip	
T	torque	Nm
ΔT	drag or idling torque	Nm
V	translational velocity	m/s
ΔV	constrained velocity	m/s
δ	angle of steering	rad
η	efficiency	
μ	coeff. of engagement	
ω, Ω	rotational velocity	rad/s
ρ	air density	kg/m^3
$\dot{x}$, dx/dt	time derivative (any variable)	any/s
{ }	column vector	
[ ]	matrix	

subscripts

1... *j*	identifier of shaft
a	aerodynamical
constr	constraint action
d	special identifier of drive wheel
h	special identifier of friction
inert	inertial action
mod	modified (due to losses)
pc	special identifier of planet carrier
pw	special identifier of planet planet wheel
sw	special identifier of sunwheel

Differential: A mechanical system with two rotational degrees of freedom, where the gears are mostly arranged as a planetary system.
Limited slip differential: A differential, where the internal relative motion is subjected to torque losses.
Virtual Differential: A Generalized Algorithm for simulation of speed and torque distribution, thus also power flow, in a differential.
Lateral Dynamics of Vehicles: Dynamics of vehicle handling, focusing on lateral translation and yaw.

3. DESCRIPTION OF A NEW APPROACH FOR ANALYSIS OF DIFFERENTIALS

3.1. Geometrical interpretation gear-tooth losses

At modelling of the differential the influence of gear tooth losses can be considered. Here, the loss of free torque can be given as follows: $T=F_t\ r_\omega$, where F_t denotes the tangential contact force component at pitch radius r_ω.
Thus, in Figure 3.1 is an illustration of a pair of teeth in contact at one of the end points of the line of contact.

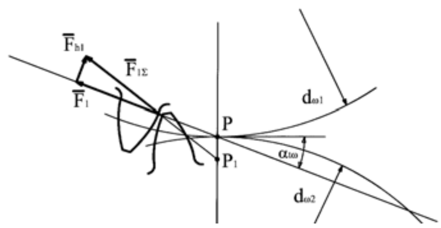

Fig. **3.1.** Loss due to friction at gearing, where, $\alpha_{t\omega}$ - pressure angle; $d_{\omega1}$, $d_{\omega2}$ - denote the diameters of the pitch circles of the driving and the driven gears, respectively; $\overline{F_1}$ - the theoretical loss free normal contact force, $\overline{F_{1\Sigma}}$ - the resultant contact force in the friction-related case, $\overline{F_{h1}}$ - the friction force; P - the pitch point, P_1 - the force pole, where the tangential force component appears to act.

The distance between P and P_1 is called the force pole offset and is denoted Δr_{ω}. In the case of friction, the magnitude of torque on the driving (subscript 1) and the driven (subscript 2) wheels can be expressed as:

$$T_1=F_t(r_{\omega 1} + \Delta r_{\omega}),\quad T_2=F_t(r_{\omega 2} - \Delta r_{\omega}), \qquad (1)$$

where Δr_{ω} is the force pole offset due to friction in the gear mesh. If either one of the wheels is internally geared, the corresponding pitch radius should be considered to be negative.

The relationships in Equation 1 may be transformed to an expression for the traditional efficiency η, where subscript 2 then identifies the internally geared wheel, if any, which calls for the lower minus sign:

$$\eta \approx 1 - \Delta r_{\omega} \left(\frac{1}{r_{\omega 1}} \pm \frac{1}{r_{\omega 2}} \right). \qquad (2)$$

The problem of the efficiency of epicyclical gear trains considering the influence of the number of teeth was an early stage treated by Jakobsson (1966).

3.2. Virtual shaft extension

This concept at power flow analysis has earlier been used by Mägi in his dissertation in 1974, Figure 3.2. is important in the description of non-parallel shaft gearing, allowing generalised definition of common positive sense for speeds and torques.

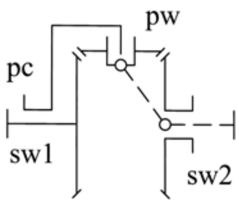

Fig. 3.2. Virtual shaft extensions (broken lines) for bevel wheel planetary system, where pc is the planet carrier, pw is the planet wheel and sw1/sw2 are sun wheels.

Speeds and torques of all externally visible real and extended shafts, as shown in Figure 3.2, might be assigned a common positive sense.

3.3. Lagrange multiplier approach

By the utilization of the Lagrange multiplier theorem (Mägi et al., 1998) the system constraint Jacobian matrix is of fundamental importance. This can easily be compiled so that it fully defines the kinematics of the model. This approach implies that originally all shafts of a differential or a planetary system in general may rotate independently. Existing interconnections, as gear meshes, introduce constraints to the motion.

Further, the constraint approach implies in the present context that the difference between the relative peripheral pitch diameter speeds in each gear mesh between two mating gear wheels, ΔV, is zero. For instance, an internal motion constraint in the system depicted in Figure 3.2, contributes to the Jacobian matrix for mesh j between shafts *pw* and *sw1* by the following:

$$\Delta V_j = (\omega_{pw} - \omega_{pc}) r_{pw} + (\omega_{sw1} - \omega_{pc}) r_{sw1} = 0, \qquad (3)$$

where r is the pitch radius of the wheel, r_{pw} denotes the left planet wheel that mates r_{sw1} and ω is the rotational speed of the shaft in consideration. r_{pw} denotes the left planet wheel that mates r_{sw1}. ΔV is the (vanishing) difference in relative pitch circle velocities at the mesh considered.

Arranging all differences of relative velocities, ΔV_j, to a vector $\{\Delta V\}$ and velocities of all shafts to another vector $\{\omega\}$, the compatibility at all constraints could be summarised from all equations of the type shown in Equation 3 to:

$$\{\Delta V\} = [J_C]\{\omega\} = \{0\}, \qquad (4)$$

where $[J_C]$ is the Jacobian matrix of the internal motion constraints in the system, containing various pitch radii as matrix elements. As Equation 4 equals zero, all radii may be premultiplied by some constant, allowing the radii to be replaced by the numbers of teeth of the wheels considered. The contribution T_{constr}, from the constraint forces, F_t, or shorter just F, to the torque equilibrium for each shaft is according to the Lagrange multiplier theorem:

$$\{T\}_{constr} = [J_C]^T \{F\}. \qquad (5)$$

In full dynamics situations, the inertial effects must be included. The change of rotational speeds, i. e., rotational acceleration, is in the sense of d'Alembert equivalent to the action of an external torque $T_{inert} = -J_P \dot{\omega}$, where J_P is the polar mass moment of inertia of the particular subsystem, and $\dot{\omega}$ is its angular acceleration. For all rotating elements the inertial torque vector is:

$$\{T\}_{inert} = [J_P]\{\dot{\omega}\}, \qquad (6)$$

where $[J_P]$ is the diagonal matrix of all polar mass moments of inertia and $\{\dot{\omega}\}$ is the vector of all angular accelerations.

The total fully dynamic but loss free equilibrium is then given by:

$$\begin{gathered}\{T\} + \{T\}_{constr} + \{T\}_{inert} = \{0\} \rightarrow \\ -\{T\}_{inert} - \{T\}_{constr} = \{T(t)\},\end{gathered} \qquad (7)$$

where $\{T\} = \{T(t)\}$ is the prescribed time dependent external torque vector. The constraint conditions, eq. 3.4, may be differentiated once, yielding:

$$\pm [J_C]\{\dot{\omega}\} = \{0\}, \qquad (8)$$

All equilibrium and compatibility equations, eq. 4 through eq. 9 could now be collected to form a common set of equations:

$$\begin{bmatrix} [J_P] & [J_C]^T \\ [J_C] & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{\omega}\} \\ \{F\} \end{Bmatrix} = \begin{Bmatrix} \{T(t)\} \\ \{0\} \end{Bmatrix}, \qquad (9)$$

The Lagrange multiplier approach eliminates the need for the detailed derivation of torque equilibrium equations.

The simplifications, described enabled the research to discuss the modules of the model in a systematic way. Thus, we can also complete the generalised model of the vehicle, which contains different modules. The model describes the distribution of the energy flow.

4. THE CHARACTERISTICS OF A VEHICLE WITH THE LIMITED SLIP DIFFERENTIAL

4.1. Tractive effort characteristics

At any radii of cornering, the rotational velocity of the driving wheels of a vehicle with a common differential is adjusted to the steering radius. The application of a planetary gear train (locked differential) leads to resistance at cornering. As a matter of fact, the coefficient of efficiency of the common differential is relatively high. Thus, in case the common differential is applied the torques of the driving wheels are approximately equal. In a loss-free case not considering gear ratio at differential, a certain correlation is noticed: $T_{in} = T_{out1} +$

T_{out2} , where: T_{in} denotes the *input* moment of the differential gear train, however, T_{out1} and T_{out2} are the output torques on the corresponding wheels (Figure 4.1). Besides, $T_{min} = Gf_{\min} r_d$ where: G is the gravitational force on the driving wheel and f_{min} denotes the coefficient of friction between the driving wheel (radius r_d) and the road surface calculated on the wheel with a lower value of friction, on the assumption that an equal force of gravitation has an impact on the driving wheels.

The highest torque in case of a common differential can be given as follows: $T_{in}=2T_{min}$.

Here, the locked differential has taken the form of a planetary gear train as a result of locking, thus enabling the transmission of a higher torque on the road:

$$T_{in}= T_{min} + T_{max} = r_d\,(Gf_{\min}+ Gf_{\max}), \qquad (10)$$

where: T_{max} denote the torque of the wheel with a higher value of friction and the coefficient of friction $f_{\max}$ on the road surface. Here, the driving wheels with equal rotational velocities provide a disadvantage at cornering.

As a rule, in most street and road vehicles, the differential with a relatively high coefficient of efficiency has been widely applied. In extreme situations (e.g. $f_{\min} << f_{\max}$), it is essential to increase the torque on the driving wheels. Here, we have tried to find a reasonable compromise between the increasing tractive effort and understeering of vehicle. To achieve that, we have tried to limit the relative mutual rotational velocity of the driving wheels.

Theoretically, the torque ratio at differential can be expressed by formula (11):

$$k = \frac{T_2}{T_1}\,, \qquad (11)$$

where T_2 and T_1 denote the torques of each drive wheel at the beginning of the beginning switch-off of the brakes.

In case of the application of the limited slip differential, the highest torque value can be expressed as the following correlation: $T_{in}=T_{min}\,(1+ k)$.

In case of jeeps (off-road vehicles) that are driven on rough roads, higher torque values are needed. It is possible to obtain higher torque values by decreasing the cornering abilities.

A. Torm (1963) has carried out tests to study the limited slip differentials in tractors. According to the test results, a differential with the torque ratio 2,0 is not likely to increase the cornering radius (tractor DT-20). In case a trailer is used in a cultivated and a stubble field, with the maximum angle of the front tyres, the cornering radius will increase by 10-20% and the total motion resistance will grow by 11-17%. However, it is not reasonable to apply a differential with a torque ratio over 2,5…3,0.

By Mägi et al., (1998) the Lagrange multiplier approach has been applied, which eliminates the need for detailed torque equilibrium equations. This is a considerable simplification of the planetary gear train analysis.

The correlation of the velocities and the torques of the shafts (rotating elements) of each transmission unit can then be automatically be formulated, presented in the form of a matrix and solved by the computer program. This will enable us to calculate the values of the torques and velocities of each shaft in the whole system.

The torque and the speed losses can be calculated by the Lagrange multiplier technique. The transmission systems with more than one input and/or an output shaft as well as the epicyclical trains can be calculated. Besides, the possible over-constrained elements of the transmission system can be detected.

4.2. Steering characteristics

Below, the characteristics of the road vehicle at cornering at low velocities have been examined (with no centrifugal force). At low speeds, a simple relation between the direction of motion and the steering wheel angle has been noticed. The prime consideration for the design of the steering system is the minimum tyre scrub at cornering. Therefore, at cornering all tires should be in pure rolling without lateral sliding. To satisfy this requirement, the wheels should follow the curved path with different radii originating from a common centre C (Figure 4.1). The steer angles δ_o and δ_i should satisfy the relationship:

$$cot(\delta_o) - cot(\delta_i) = B/L\,, \qquad (12)$$

where, the subscripts o and i denote outer and inner wheels at cornering. The steering geometry that satisfies the above equation is usually referred to as the Ackermann steering geometry and is valid as a theoretical reference case where sideslip of wheels is disregarded (Wong, 1993).

Model treatment and influence to the steering characteristics have been handled in Resev (2002).

In Figure 4.1, C is also an instantaneous centre and Ω is the rotational velocity relative to it. The steering angles of the front wheels δ_o and δ_i have been approximated to the longitudinal axis of the vehicle used in the calculations: $\delta = 0.5(\delta_o + \delta_i)$.

According to the value of the under steer coefficient Resev (2002) or the relationship between the slip angles of the front and the rear tires, the steady-state handling characteristics may be divided into the three categories: neutral steering, understeering and oversteering.

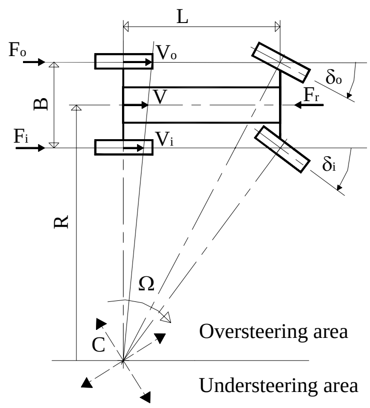

Fig. 4.1. The power and steering characteristics of the vehicle. Where, B is the track (or tread) of the vehicle, L is the wheel base, F - force, V - velocity, whereas the subscripts o (outer) and i (inner) denote the relative displacement of the instantaneous centres C. r denotes the resistance and Ω - rotational velocity of the vehicle.

In the design of the differential it is essential to follow the principles:

- The differential is expected to provide the vehicle with the utmost neutral steering qualities.
- The characteristics that influence the steering of the car should change the steering for more neutral so that it contributes to the steering qualities.

Constructively can be a shifted symmetrical limited slip acts like an asymmetrical differential in a curvilinear trajectory.

4.3. Components of the virtual differential model

The systems studied have been composed of the different model components. In the description of the model components we need an analysis of the general process of motion. The vehicle moves along the non constant radius by the changing angle of the front wheels.

The lateral dynamics of the limited slip differential can be divided into certain stages. As for the characteristics of the differential, the most significant factor here is the constraint on the inner slip.

This constraint may have a variable or constant impact on the torque ratio. In case it is variable, it can be load- or velocity proportional. Let us take a limited slip differential, which is load proportional. The difference between the velocities of the drive wheels: $\Delta V = V_o - V_i$.

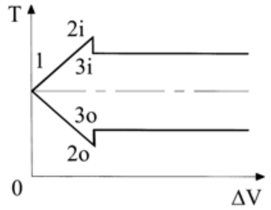

Fig. 4.2. The torque ratio at differential of the motion process on a curvilinear trajectory. Where, *T* – the drive wheel torque, *ΔV* - the relative velocity of the drive wheels.

The lateral dynamics of a vehicle equipped with a differential of the kind undergoes the following processes:

1. The transition from the curvilinear trajectory to the linear one.
2. The transition from the linear trajectory to the curvilinear one with a variable radius:

- The stage of the relative small torque of the driving wheels– the differential is still locked (1-2i or 1-2o, Figure 4.2). The vehicle is moving along a curve - the rotational velocities of its driving wheels are equal ($\omega_\iota = \omega_o$). The inner wheel of the curve slips back in relation to the road surface, whereas the outer wheel slips forward in the direction of motion. However, both driving wheels slip back as a result of the motional resistance.
- The intermediate stage – relative slip is noted in the friction elements of the differential (2i-3i or 2o-3o Figure 4.2). Here, we can define the unlocking phase of the differential, which is characterized by the transition from the static friction to the dynamic friction. As a rule, the transition is gradual, dependent on the surfaces of friction.
- The stage of the large angle of the front wheel – the differential has been unlocked (at point 3i or 3o, Figure 4.2). The further increase in the torque of the driving wheels will not bring about an increase in the relative torque ratio.

These stages can be interpreted as modules for the vehicle movement models.

For the modelling of the limited slip differential we need speed and load characteristics of the vehicle as the initial parameters. The general equations of the limited slip differential for the simulation of the lateral dynamics of the vehicle (Figure 4.1) can be correspondingly divided into equilibrium, compatibility and constitutive relations.

The equilibrium of the force and torque has been generally shown as $\Sigma T = 0$, $\Sigma F = 0$.

We can overcome the resistance at constant movement by making use of the tangential force of the driving wheels:

$$F_r = F_i + F_o , \qquad (13)$$

As a rule can be given generally as:

$$m\,{}^{dV}\!/_{dt} = F_i + F_o - F_r , \qquad (14)$$

where *m* denotes mass of the vehicle and F_r is the resistance of motion.

The compatibility of the geometrical relations can be given as: for the outer and inner drive wheels subsequently:

$$\begin{aligned} \omega\ r(1-s_o) &= V_o \\ \omega\ r(1-s_i) &= V_i , \end{aligned} \qquad (15)$$

where *s* denotes slip. According to Figure 4.1, for the curvilinear motion of the vehicle can be given as:

$$\frac{V_o}{V_i} = \left(\frac{R+0.5B}{R-0.5B}\right) = \frac{1-s_o}{1-s_i} \qquad (16)$$

The constitutive relations of the limited slip differential for the tangential force relatively of the drive wheels can be expressed as the following analytical approximation:

$$F_{i,o} = \frac{mg\mu}{2\pi} \arctan(20s_{i,o}). \qquad (17)$$

The motion resistance:

$$F_r = \pm k_r mg + k_a \frac{\rho A}{2} V^2 , \qquad (18)$$

where k_r denotes the rolling resistance, k_a is the coefficient of aerodynamic resistance, A is a vehicle front area, μ is coefficient a wheel engagement on road and *g* is acceleration due to gravity. The overwritten relations include the main parameters have been modelled of the design differential. The motion process has simulated by MatLab.

At the stage of modelling it is important to follow the velocity parameters and record the load characteristics, thus the power flow models from the engine to the driving wheels could be improved.

CONCLUSION

The basic equations for the description of the module of the virtual differential are elaborated. Based on these equations, the module system of the vehicle model could be derived.

The model of the differential constitutes a part of the module system of the vehicle model.

The proposed model enables to observe and guide the distribution of the energy flow.

The model enables us to follow and change the parameters of vehicle involved in.

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