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DETECTION OF CAVITATION IN PUMPS THROUGH HIGHER ORDER DERIVATIVES

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Abstract: *The common methods of condition monitoring are typically sufficient for estimating the condition of machines. Sometimes there are however weak signals in vibration that do not contribute much to typical condition monitoring measurement parameters, like often happens in the case of cavitation or for example in the case of early bearing faults. In such case differentiation of vibration signal may help.*
Key words: Differentiation, vibration, cavitation.

1. INTRODUCTION

The methods of early diagnosis of machine faults are constantly being developed and enhanced. One of the main goals of machine diagnostics is the discovery of faults in their early stages, well before they lead into failure.
The main method used in diagnostics is vibration-based approach, which allows evaluating machine's condition as a whole and also condition of its elements separately. The magnitude of vibration is mostly described by deflection, velocity and acceleration, i.e. $x \equiv x^{(0)}$, $dx/dt \equiv x^{(1)}$ and $d^2x/dt^2 \equiv x^{(2)}$.
The aforementioned parameters have alongside their benefits also some insufficiencies. In some cases faults are difficult or even impossible to detect using these parameters. This is often the case with cavitation and this is where differentiated signals become useful.
Perhaps the biggest problem in detecting cavitation is that it's not always accompanied by increase of overall vibration levels. Sometimes the rotation speed is small and although machine is cavitating vibration levels don't change. Often the vibration amplitudes created by cavitation are too small to increase the vibration level of machine. In many cases listening to vibration signal may hint the presence of cavitation, but measurements don't. Prof. Sulo Lahdelma (University of Oulu) has shown in his works (Lahdelma, 1995, 1996, 1997, 2002) that such phenomena can be discovered with the help of higher order differentiation.
An algorithm has been developed in the Department of mechatronics of TTU based on the work of S.Lahdelma to calculate higher differentiation (and integration) orders of vibration signals and to investigate cavitation. Also a test rig has been built to run tests on smaller pumps with different setups and conditions. The idea to investigate pumps was suggested to us by Prof. S.Lahdelma who had used this method for cavitation detection in turbines.

2. CALCULATION OF DERIVATIVES

Mathematical description of vibrations is often done using complex exponentials. Differentiation can be expressed by the base function is in the form of

$$x(t) = Xe^{i\omega t} \tag{1}$$

The derivative $x^{(\alpha)}$ of a function (1) can be defined as

$$x^{(\alpha)} = \omega^{\alpha} Xe^{i(\omega t + \alpha\pi/2)} \tag{2}$$

where
parameters α (any real number), ω and X are constants, e is a Napier number, t is a time variable.
From formula (2) it can be seen that in the derivative the quantity X is multiplied by ω^{α} and phase angle varies proportionally to the order of derivative α that is by $\alpha\pi/2$.

Taking into account that

$$e^{i\frac{\alpha\pi}{2}} = \left(e^{\frac{i\pi}{2}}\right)^{\alpha} = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{\alpha} = i^{\alpha},$$

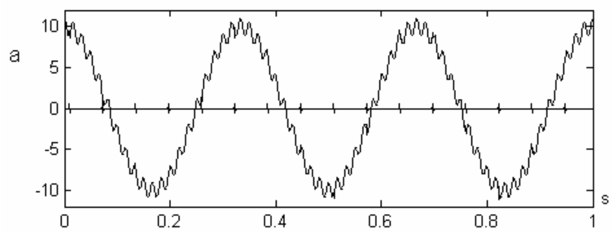
we can write formula (2) as follows

$$x^{(\alpha)} = e^{i\frac{\alpha\pi}{2}} \omega^{\alpha} Xe^{i\omega t} = i^{\alpha} X_{\alpha} e^{i\omega t} = (i\omega)^{\alpha} x = \hat{x}_{\alpha} e^{i\omega t}$$

where

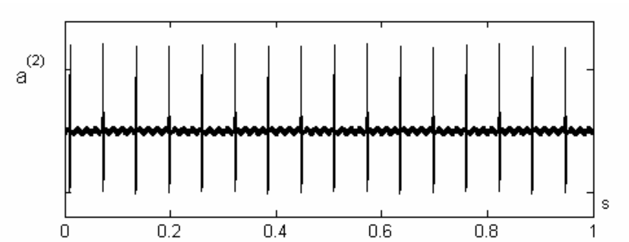
$$X_{\alpha} = \omega^{\alpha} X \quad \text{and} \quad \hat{x}_{\alpha} = (i\omega)^{\alpha} X.$$

Let us look at a simple example of how useful derivatives can be in detecting weak but intensive signal components. We present a simulated signal where we have dominating vibration of 3 Hz with the amplitude of 10 units, which is complimented by higher frequency vibration of 60 Hz with amplitude of 1 unit. Inside this vibration we have hidden an impulse signal that has amplitude 0.5 units and occurs about 6 times during one period of dominating vibration of 60 Hz. The resulting signal is shown in figure 1.



Fig, 1. Original signal with impulses shown separately

The impulse signal that we've planted in our vibration is shown separately. As you can see there is almost no visible indication of impulses in the resulting signal. Higher frequency component of 60 Hz is completely dominant.



Fig, 2. Second time derivative $a^{(2)}$ of the original signal

Although the pulses are weak and occur at long intervals, the rate of change of their amplitude is quite intense and become emphasized upon differentiation of the signal.
Now when we take the second time derivative of the original signal (figure 2) we immediately see how impulses are significantly emphasized and the other frequencies are diminished.

3. CAVITATION DETECTION PROBLEM

In the previous work on cavitation in pumps (Teder, 2003) the emphasis was on the detectability of cavitation via the use of derivatives and the importance of the chosen frequency range. After a meeting with Prof. Lahdelma and analysis of results presented in aforementioned paper the decision was made to *increase the frequency range* and to use *short-time peak value* averaging instead of additive averaging. The idea here is to make the signal more sensitive to cavitation.
It was mentioned previously that weaker signals in vibration do not contribute much to typical condition monitoring measurement parameters. Let us look into that briefly. The most typical condition monitoring parameter would be vibration speed in mm/s. This is by far most used vibration parameter, because it is proportional to vibration energy and typically gives most adequate picture about machine condition. The main vibration standard is ISO 2372 and it is based on the assumption that machine faults are determined through vibration in the frequency range of 10-1000 Hz and the machine condition is evaluated by the root mean square (rms) of vibration velocity.
In some cases faults or conditions are difficult or even impossible to detect using velocity parameter. One of such conditions is cavitation. Cavitation can be characterized as a phenomenon that is of intense nature (many impulses occur during a short period of time). In such case even the acceleration impulses are often weak, but the variation of acceleration is intense and the values of derivatives of acceleration are significantly higher compared to non-cavitating signal.

4. TEST MEASUREMENTS AND ANALYSIS

We will now examine results of a tests conducted on a small 1.1 kW cold water pump in our laboratory. By variating incoming or outgoing water flow we were able to create noticeable cavitation effect at speeds beginning around 20 Hz. We verified the presence of cavitation with a stethoscope. Without the use of stethoscope there was no audible difference in the pump noise. The tests were carried out at several different speeds.
Frequency range for these tests was increased to 5000 Hz assuming there might be an increase in amplitudes at higher frequencies. Previously used frequency range was up to 2000 Hz and comparative test were done using that range. Since vibration is a more or less variating process a standard *additive averaging* is typically used when making measurements to achieve more adequate results. For cavitation measurements however it might be better to use *short-time peak value averaging* (not more than 5 averages). This was a suggestion made by Prof. Lahdelma based on his experience. The idea here is that cavitation is a process where there are strong rapid bursts in short period of time. When listening with stethoscope one can hear several pops in short time periods.
Measurements results are shown in tables 1-4 in form of comparison between cavitating and non-cavitating signal's overall rms values. For detection of cavitation we used first and second derivative of acceleration signal. Acceleration results are shown for comparison.

Frequency	**v rms** (mm/s)	**a rms** (mm/s^2)	**a(1) rms** (mm/s^3)	**a(2) rms** (mm/s^4)
20 Hz cavitation	0,43	316	2,38E+06	2,30E+10
20 Hz no cav.	0,33	240	1,84E+06	1,80E+10
Ratio	**1,3**	**1,3**	**1,3**	**1,3**
28 Hz cavitation	2,24	1657	1,20E+07	1,00E+11
28 Hz no cav.	2,12	457	2,30E+06	2,30E+10
Ratio	**1,1**	**3,6**	**5,2**	**4,3**
37 Hz cavitation	4,41	2891	1,93E+07	1,80E+11
37 Hz no cav.	4,48	1386	8,30E+06	7,90E+10
Ratio	**0,98**	**2,1**	**2,3**	**2,3**
47 Hz cavitation	2,07	4438	3,65E+07	3,60E+11
47 Hz no cav.	1,8	735	4,27E+06	4,20E+10
Ratio	**1,2**	**6,0**	**8,5**	**8,6**

Table 1. Measurements in frequency domain of 2000 Hz

Frequency	**v rms** (mm/s)	**a rms** (mm/s^2)	**a(1) rms** (mm/s^3)	**a(2) rms** (mm/s^4)
20 Hz cavitation	0,61	476	3,97E+06	3,90E+10
20 Hz no cav.	0,49	328	2,60E+06	2,60E+10
Ratio	**1,2**	**1,5**	**1,5**	**1,5**
28 Hz cavitation	2,39	2563	1,92E+07	1,60E+11
28 Hz no cav.	2,18	554	3,58E+06	3,60E+10
Ratio	**1,1**	**4,6**	**5,4**	**4,4**
37 Hz cavitation	4,45	4196	2,95E+07	2,70E+11
37 Hz no cav.	4,68	1787	1,28E+07	1,20E+11
Ratio	**0,95**	**2,3**	**2,3**	**2,3**
47 Hz cavitation	2,45	6666	5,76E+07	5,70E+11
47 Hz no cav.	1,88	945	6,55E+06	6,50E+10
Ratio	**1,3**	**7,1**	**8,8**	**8,8**

Table 2. Measurements in frequency domain of 2000 Hz using peak value averaging.

Frequency	**v rms** (mm/s)	**a rms** (mm/s^2)	**a(1) rms** (mm/s^3)	**a(2) rms** (mm/s^4)
20 Hz cavitation	0,44	1411	3,39E+07	8,50E+11
20 Hz no cav.	0,33	377	7,37E+06	1,80E+11
Ratio	**1,3**	**3,7**	**4,6**	**4,7**
28 Hz cavitation	2,24	3595	7,86E+07	1,90E+12
28 Hz no cav.	2,12	607	1,01E+07	2,50E+11
Ratio	**1,1**	**5,9**	**7,8**	**7,6**
37 Hz cavitation	4,41	5394	1,13E+08	2,80E+12
37 Hz no cav.	4,48	1552	1,85E+07	4,20E+11
Ratio	**0,98**	**3,5**	**6,1**	**6,7**
47 Hz cavitation	2,11	10089	2,23E+08	5,60E+12
47 Hz no cav.	1,85	1216	2,46E+07	6,20E+11
Ratio	**1,1**	**8,3**	**9,1**	**9,0**

Table 3. Measurements in frequency domain of 5000 Hz

Frequency	**v rms** (mm/s)	**a rms** (mm/s^2)	**a(1) rms** (mm/s^3)	**a(2) rms** (mm/s^4)
20 Hz cavitation	0,62	1870	4,48E+07	1,10E+12
20 Hz no cav.	0,49	470	8,59E+06	2,10E+11
Ratio	**1,3**	**4,0**	**5,2**	**5,2**
28 Hz cavitation	2,40	5317	1,15E+08	2,90E+12
28 Hz no cav.	2,18	749	1,28E+07	3,10E+11
Ratio	**1,1**	**7,1**	**9,0**	**9,4**
37 Hz cavitation	4,46	8057	1,70E+08	4,20E+12
37 Hz no cav.	4,68	2028	2,56E+07	5,60E+11
Ratio	**0,95**	**4,0**	**6,6**	**7,5**
47 Hz cavitation	2,52	14734	3,21E+08	8,00E+12
47 Hz no cav.	1,88	1491	2,95E+07	7,40E+11
Ratio	**1,3**	**9,9**	**10,9**	**10,8**

Table 4. Measurements in frequency domain of 5000 Hz using peak value averaging.

From the results we immediately see the differences in ratios of different signal types. The measurements of vibration velocity do not differ much, just as expected.

In case of acceleration there is some notable difference but as experience tells us this ratio is often insufficient, especially at lower speeds. Results from this test session are actually surprisingly good for acceleration.

If we now look at $a^{(1)}$ and $a^{(2)}$ signals (first and second derivative of acceleration) we see a significant difference between the cavitating and non-cavitating signal. Such increase in the overall vibration level of $a^{(1)}$ and $a^{(2)}$ signals sends a clear message that machine is behaving differently. With higher speeds and stronger cavitation, using 5000 Hz frequency domain, the ratio is between 6-10 which is a very good result. With lower speeds the ratio is over 4 which is good enough to get conclusive result.

As you can see, using higher frequencies also increases detectability of cavitation. In previous work we went up to 2000 Hz. In this new set of tests we used 5000 Hz. The best way to see

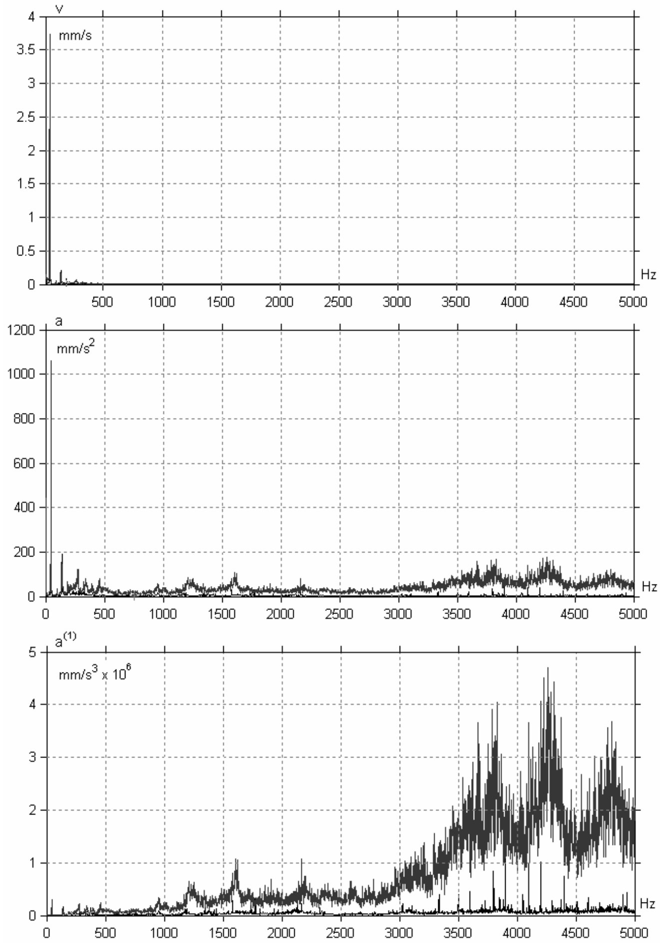


Fig. 3. Vibration speed v, acceleration a and first derivative $a^{(1)}$ of acceleration of cavitating (gray) and non-cavitating(black) pump.

the difference between signals is to look at fig. 3., the spectrum of signal $a^{(1)}$. Frequency spectrum of cavitation signal is in gray and non-cavitating signal's spectrum is in black. The difference is quite impressive.

We also get interesting results comparing standard additive averaging with short-time peak value averaging. There's not much difference in frequency domain of 2000 Hz. However in 5000 Hz measurements we get about 10-20% increase in ratios. This is fairly good result but more tests need to be run to determine how effective peak value averaging is with different measurement setups.

5. CONCLUSION

The tests for cavitation detection conducted on a small cold water pump using first and second derivative of acceleration have given very promising results. We have concluded that good estimation of cavitation can be achieved with the first and second derivatives $a^{(1)}$ and $a^{(2)}$ of acceleration using commonly available analyzers. For better results wider frequency domain can be used including higher frequencies plus short-time peak value averaging.

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