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DETERMINATION OF THE NONCENTROIDAL BODY BEHAVIOUR

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Abstract: *The main point of this work is investigation of free oscillations of a noncentroidal body. During oscillation a translation process is possible. The translation movement is determined by the ratio of the inertia vector against the friction force. The results of the investigation are envisaged to be used for simplifying the evaluation of certain technological equipment with the help of MATLAB Simulink software. The energetic method has been used for defining free oscillation and translation movement. A body having the shape of a circumference was analysed, but any other forms in real technological equipment can also exist.*

Key words: *Noncentroidal body, rolling and sliding motion, friction, computer simulation.*

1. INTRODUCTION

The motion of a rigid body constrained to rotate about a fixed axis which does not pass through its mass centre is called noncentroidal rotation and the body itself – noncentroidal.
A body can roll without or with sliding. It is important to formulate the conditions, when sliding starts. When sliding is impending, the friction force F reaches its maximum value F = μ_SN, and may be obtained from N, where μ_S is the coefficient of static friction, N – normal reaction.
When the body rolls and slides at the same time, the relative motion exists between the point of the disk, which is in contact with the ground, and the ground itself, and the force of friction has the magnitude F = μ_KN, where μ_K is the coefficient of kinetic friction.

2. VECTORS OF NONCENTROIDAL BODY

Investigating motion, when mass centre G does not coincide with its geometric centre O, we determine the acceleration of mass centre in terms of angular acceleration and the angular velocity of the body, i.e. we use the relative acceleration formula (Beer&Johnston, 1962):

$$\bar{a} = a_G = a_O + a_{(G/O)} = a_O + a_{(G/O)n} + a_{(G/O)t} \qquad (1)$$

where a_O – the magnitude of acceleration of the geometric centre,
$a_{(G/O)n}$, $a_{(G/O)t}$ – the normal and tangential components of relative acceleration $a_{(G/O)}$.

In this case (Fig. 1.) the motion of mass centre G and the rotation of the body about O are independent.
From the figure it is seen that, if the initial state (conditions) of the body movement corresponds to that in the diagram, the influence of the projections of the normal and tangential components of relative acceleration does not stimulate sliding – the normal reaction is increased, whereas the influence of a_O is decreased.

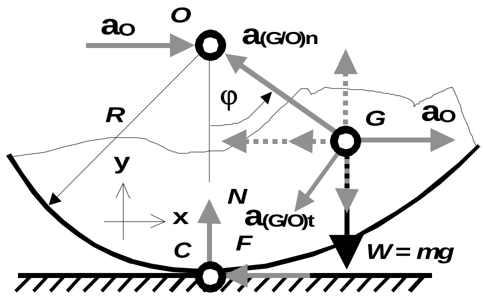

Fig. 1. Vectors on noncentriodal body when rolling starts

We shall determine the components of acceleration of mass centre G

$$a_O = R\frac{d^2\varphi}{dt^2} \qquad (2)$$

$$a_{(G/O)t} = OG\frac{d^2\varphi}{dt^2} \qquad (3)$$

$$a_{(G/O)n} = OG\left(\frac{d\varphi}{dt}\right)^2 \qquad (4)$$

where R – radii of body,
OG – distance between centre of rotation and centre of mass,
φ – angular displacement.

If the body has turned in such a way (Fig. 2.), that its mass centre is on the left from the support point C, the influence of the components of relative acceleration, on the contrary, stimulates sliding, the normal reaction is decreased, the influence of acceleration a_O is increased and, at definite numerical values, sliding begins.

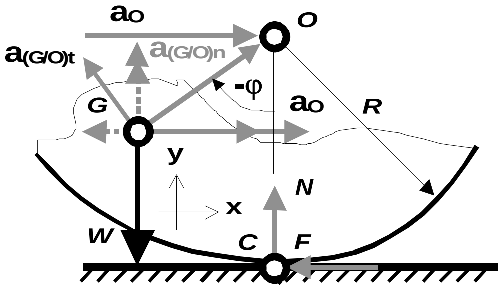

Fig. 2. Vectors of noncentroidal body when sliding starts

When it is not known whether the body slides or not, it should be assumed that the body rolls without sliding. If inertia vector F_{in} is found smaller than, or equal to, $\mu_S N$, the assumption is proved correct.
From Fig. 1 we can write

$$F_{in} = m(a_O - a_{(G/O)n}\sin\varphi - a_{(G/O)t}\cos\varphi) \qquad (5)$$

and

$$N = m(g + a_{(G/O)t}\sin\varphi - a_{(G/O)n}\cos\varphi) \qquad (6)$$

Sliding starts when

$$F_{in} > \mu_S N \qquad (7)$$

It allows us to formulate start conditions of sliding in the terms of parameters of body and rolling motion

$$[R - OG(\cos\varphi + \mu_S \sin\varphi)]\frac{d^2\varphi}{dt^2} - \\ -[OG(\sin\varphi - \mu_S\cos\varphi)](\frac{d\varphi}{dt})^2 > \mu_S g \qquad (8)$$

It is very important to investigate the possibility of simplifying the starting condition of sliding (8). As it was stated above, first of all it should be assumed that the disk rolls without sliding. The energy method should be applied for finding the equation of free rolling oscillations of the body and then analysed, by simulating it.

3. EQUATION OF ROLLING OSCILLATIONS

When dissipation of energy in a system is small, the system is called conservative. In our conservative system the energy in free oscillations is partly kinetic and partly potential. The kinetic energy is stored in the mass by virtue of its velocity, whereas the potential energy is stored in the form of work done against a force field such as gravity. The total energy, however, is constant, and the rate of change of the total energy must therefore be zero (Thomson, 1965).

$$\frac{d}{dt}(T + U) = 0 \qquad (9)$$

where T - kinetic energy of the body,
U – potential energy of the body.

Furthermore, it is evident that the maximum kinetic energy must equal the maximum potential energy.
In determining the kinetic and potential energies (Fig. 3.), it must be taken into account that both translation and rotation take place.

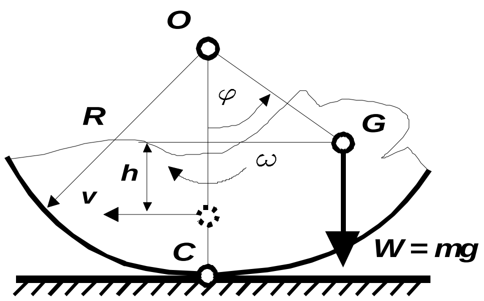

Fig. 3. Body model for determining energies

The kinetic energy may be written as

$$T = \frac{1}{2}[m(R - OG)^2 + I_G](\frac{d\varphi}{dt})^2 \qquad (10)$$

where m – mass of the body,
I_G - moment of inertia about of the mass centre .

The potential energy referred to its position at the lowest point

$$U = mgOG(1 - \cos\varphi) \qquad (11)$$

Substituting (10), (11) in equation (9)

$$[m(R - OG)^2 + I_G)\frac{d^2\varphi}{dt^2} + mgOG\sin\varphi = 0 \qquad (12)$$

During rolling the oscillations of the body energy are dissipated in the form of friction or some other form. The actual description of the damping force associated with the dissipation of energy is difficult (Routh, 1985). However, ideal damping models can be conceived which will often permit a satisfactory approximation. Of these the viscous damping force, proportional to the first power of the angular velocity, leads to the simplest mathematical treatment (Thomson, 1965).
Applying viscous damping in (12) it assumes the form of homogeneous second-order differential equation

$$\frac{d^2\varphi}{dt^2} + D\frac{d\varphi}{dt} + \frac{gOG}{(R - OG)^2 + \frac{I_G}{m}}\sin\varphi = 0 \qquad (13)$$

where D – viscous damping coefficient

The equation (13) will be used for analysing the starting conditions of sliding of the body by simulating it with Simulink software.
Natural frequency wN of the body

$$\omega_N \approx \sqrt{\frac{gOG}{(R - OG)^2 + \frac{I_G}{m}}} \qquad (14)$$

4. SIMULATION OF BODY BEHAVIOUR

To find out the influence of the components of relative acceleration on the normal reaction N and inertia vector Fin, free oscillations of a noncentroidal body are simulated with MATLAB Simulink software making use of the equation (13).
The block diagram of simulation is shown in Fig. 4.
The simulation block diagram envisages an analysis of the changes of the normal reaction N during the oscillations of the body, as, in compliance with the equation (7), the beginning of the sliding movement depends on it..
Scope 1 show the time diagrams of angular displacement, velocity and acceleration when the body oscillates, and they are presented in Fig. 5.
Diagram 1 represents the acceleration, diagram 2 – the velocity, and diagram 3 – the displacement.
Scope 2 shows the time diagram of N/m and a(O/G) components when the body oscillates, and they are presented in Fig. 6.
As seen from Fig. 6., the normal reaction of a noncentroidal body during oscillations has a variable component with a double frequency of the body self oscillation (natural) frequency wN.

The first task, with solving of which the investigation was continued, was simplification of the sliding start condition (8).
It is based on the fact that, by making physical experiments and making shots with a digital video camera of the movement of noncentroidal bodies, it has been found that during one half-period of free oscillations of a body there can be only one sliding. Thus, the influence of changing of the variable component of normal reaction N can be simplified. On the other hand, as the values of the friction coefficient is of random character which does not allow us to precisely determine the beginning and also the end moment of sliding, the complicated expression (8) of the beginning of sliding is of no practical value.

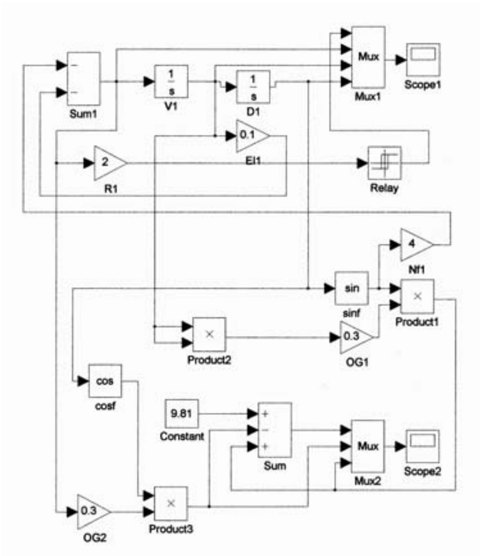

Fig. 4. Simulation block diagram for analysing free oscillations

Showing graphically and evaluating the relationship and influence of the variable components in sliding start conditions, it has been found that the sliding start condition can be sufficiently precisely determined in a simplified way by

$$R\frac{d^2\varphi}{dt^2} > \mu_s g \qquad (15)$$

The end of sliding is a controversial issue, however, physical experiments allow us to maintain that the following condition can be used for approximate evaluation.

$$R\frac{d^2\varphi}{dt^2} < \mu_s g \qquad (16)$$

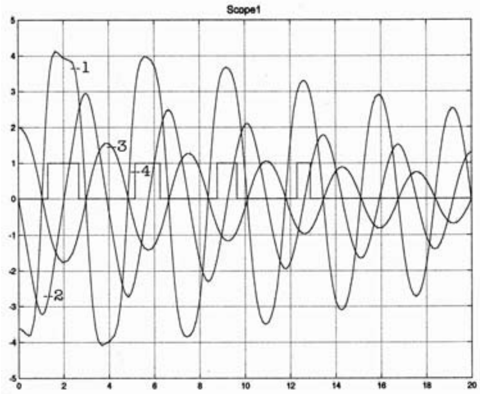

Fig. 5. Time diagrams of oscillation parameters

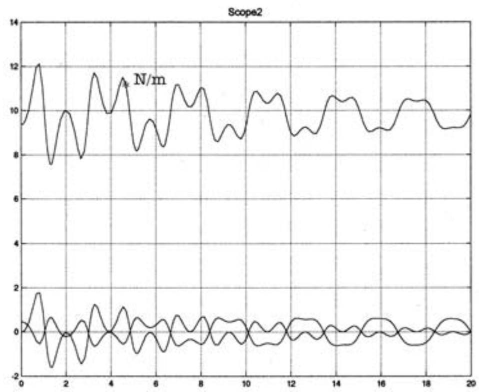

Fig. 6. Normal reaction and its component behaviour during free oscillations of the body

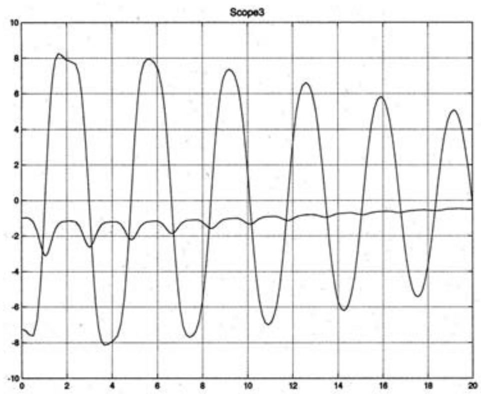

Fig. 7. Inertia vector and its component time diagrams during the oscillation process (Scope 3 not shown in Fig. 4.)

To show the time of the sliding movement in the oscillation parameter time diagram (Fig. 5.), the simulation block diagram (Fig. 4.) uses the element Relay, which forms the unity signal =1 at its output, if the condition (15) has been fulfilled, but it is removed, if the condition (16) has been fulfilled.
Thus, in this time diagram there can be analysed both the initial and final moment of the sliding movement.
The sliding movement in the time diagram (Fig. 5.) is shown by lines 4.
The behaviour of a noncentroidal body during its oscillations has also other peculiarities which can be considered. Thus, if the body is symmetrical, its sliding movement during oscillations is of a reciprocating character. However, in connection with the energy losses caused by overcoming the forces of dry friction, the amplitudes of the speed and acceleration in the previous oscillation period are larger than the amplitudes of the following period and, with this the oscillating body slides in one definite direction which depends on the starting conditions.
Quite a different scene can be observed if the noncentroidal body is asymmetric, i.e. its outer surface is formed by curves with different curve radii and the mass of the body is concentrated in several points. In this case the sliding movement can take place only in one direction, and this fact increases the efficiency of the translation of the body. The described peculiarities of the movement are shown by lines 4 in Fig. 4.
Another problem to be touched upon is the right to substitute the force of dry friction by the force of wet friction (viscous resistance, viscous damping). There have been experiments (Popov,1987) which confirm the former and determine the conditions at which it can be done.

In our research coefficient D (13) must be chosen in accordance with the results of physical research so that the character of movement in the case of dry and viscous friction would be identical.

6. DISPLACEMENT IN SLIDING MOTION

The principle of work and energy offers a tool for express evaluation of the displacement of a noncentroidal body in a sliding motion, which states that (Sautas-Little & Inman, 1999)

$$T_1 - U_{1-2} = T_2 \tag{17}$$

where T_1,T_2 – initial and final values of kinetic energy in one cycle of rolling motion of noncentroidal body when sliding exists,
U_{1-2} – work of forces done during the movement.

If the kinetic energy has changed during the period of the movement of a noncentroidal body, it has happened as a result of the body using up energy to overcome dry friction. It will be determined by equalling the changes of kinetic energy to the work done to overcome the dry friction.

The kinetic energy in the rolling motion of body is

$$T = \frac{1}{2} I_G \omega^2 \tag{18}$$

where I_G – moment of inertia of the body about the mass centre (Fig. 3.),
ω – angular velocity of the body.

Supposing that the only work in the sliding motion is done by dry friction, U_{1-2} can be expressed

$$U_{1-2} = \int_{s1}^{s2} F ds = \mu_k N s \tag{19}$$

where μ_k – the coefficient of kinetic friction,
N = mg – supposing the magnitude of normal reaction constant,
s – displacement in one cycle of sliding motion.

According to Fig. 5. and expression (17) we can write

$$\frac{1}{2} I_G \omega_1^2 - \mu_k mgs = \frac{1}{2} I_G \omega_2^2 \tag{20}$$

where ω_1, ω_2 – the values of angular velocity at the start and end time moments of sliding motion.

From equation (20) we get

$$s = \frac{I_G(\omega_1^2 - \omega_2^2)}{2\mu_k mg} \tag{21}$$

which allows us to calculate displacement at every cycle of sliding motion.

7. CONCLUSIONS

The study of the noncentroidal body behaviour in the period of its free oscillations was envisaged with the aim of studying the operation efficiency of technological equipment.

The sliding conditions depend on the interconnection of the numerical values of the inertia vector, normal reaction and friction coefficient. This fact makes it difficult, even in the case of one body, to obtain simple information on the common process of the rotation and sliding movement.

In the result of our research it was found out that adequate information on the beginning of sliding can be obtained only by studying a rolling motion where the energy losses caused by overcoming dry friction are substituted by equal losses of viscous friction.

Displacement values in a sliding movement can be evaluated by equalling energy losses of a body during one period of free oscillations to the work done by dry friction.

Experimental research of a noncentroidal body movement carried out with the help of a videocamera allows us to maintain that considerable displacement of sliding can be obtained by using an asymmetric body that is why an equation with variable structure should be used in this case.

8. REFERENCES

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