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MODIFIED CALCULATION METHOD OF TOLERANCE OF DIMENSIONAL CHAIN DEPENDENT LINK

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ABSTRACT

Industrial constructions mainly are made up of the several components which material has different physical properties. Parameters of the details and the assemblies build up the dimensional chain.
Existing dimension chain calculation theory is widely used in design process of new machines and other products. Theory can also be used in measurement analyse process for the existing product elements and it helps to analyse tolerances and limits effectiveness of a product already in use. The main goal in dimension chain solving is to fix tolerances and limits to all measurements in chain. In the process of dimension chain calculation, it's necessary to take into consideration the real work situation of chain links.
As novelty, in this work, we give the modified calculation principles of dependent dimension tolerances and limits. By calculation we are taking into account properties influence of the dimension chain single elements. Its necessary to assure the functioning of chain parts in work process (work environment of chain parts) in accordance with requirements; it's also necessary to consider the parameters of different materials used in the chain. Modern ideology is used for uncertainties calculation what which is given by GUM (Guide to the Expression of Uncertainty in Measurements).
The main goal of this article is to introduce the modified (new) method for the calculation of dependent dimension tolerance and value. In this method the dimension chain single element measurements are considered as random variables with characteristic uncertainty.
Key words: dimensional chain, dependent dimension, random variable, uncertainty

1. INTRODUCTION

In many cases industrial construction consists of components and details which have multiple measures and it measures which are composed dimensional chains. This fact cause needs to carry out additionally to strength and kinematics calculation besides to dimensional chain parameters calculation.
Theory of dimensional chain calculation (Dunajev, 1963; Aasamäe, 1976) is widely used for construction new machines and mechanisms. This theory is applied if there is a needed to carry out analyses of earlier produced products' components' measures. Goal of such analyses is to estimate used measures tolerances and limits fitness of measures used and change of product linear parameters in exploitation.
But dimensional chain calculations nevertheless as min-max or probability do not take account change of parameters which is caused by conditions of product exploitation or caused by time and structural change of product details materials. Environment temperature and moisture and pollution have especial influence to exploitation.
In this study work currently are under observation in dependent link nominal value, tolerance and limit deviations calculations taking into account systematic effect influence to chain links values. Therefore for dependent link value and tolerances finding modified calculation method is used, where dimensional chain links additionally may have systematic deviations which have ascertain uncertainties. This allows finding more accurate value for dependent link.

2. THEORETICAL GROUND FOR MODIFIED CALCULATION METHOD

Standardised values of tolerances are found as random values and summarised are factors from main components as shown in Fig. 1.

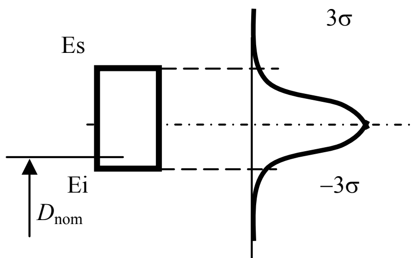

Fig 1. Tolerance structure allowing the use of normal distribution statistics

Main components causing the tolerance between its limits values Es and Ei:

1. Technological process.
2. Control instrument.
3. Change of environmental conditions.
4. Object behaviour.

Probability theory confirms that in case there exists at least four random values then their summary is more or less normally distributed (as shown in Fig.1) and this allows also the use of probability method for calculation of dimensional chain and summarise factors uncertainties to tolerance value.
Dependent link, in general case expressed as Y, value can be found through functional dependence f from dimensional chain other links parameters or input values X_i (i = 1, 2,..., N) by equation

$$Y = f(X_1, X_2,..., X_i,..., X_N) \qquad (1)$$

But input values X_i from which output value Y is depended, can be observed as values, which can depend from other values including systematic effects causing corrections. This forms

complicated functional dependency f which is impossible to be described correctly.
Functional dependence f shall be observed as combined function including dimensional chain links values and corrections, which can give essential part to dependent link value and for this uncertainty formation.
On Fig 2 there is shown dependent link movement through the influence of systematic factors.
On Fig 2 there are used next terms and symbols:

– initial parameter

– parameter having systematic factor

Es1, Es2, Ei1 and Ei2 – upper and lower deviations limits of dimensional chain links.
Es and Ei – upper and lower deviations limits of dimensional chain dependent link.
On Fig 3 is shows dependent link tolerance and uncertainty of systematic factors which to be added.
Used are Es_{sys} and Ei_{sys} as tolerance (uncertainty) limits of systematic factors.

3. ESTIMATES OF DEPENDENT LINK AND INPUT PARAMETERS VALUES

As input parameters are included dimensional chain links are included. Its values and uncertainties are found directly from technical figure through solving the dimensional chain.

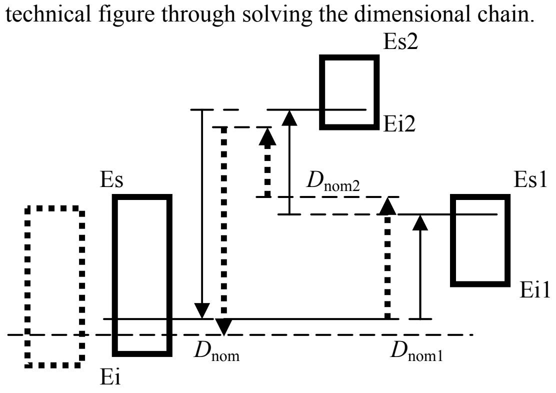

Fig 2. Dimensional chain movement (shift) caused by systematic factors

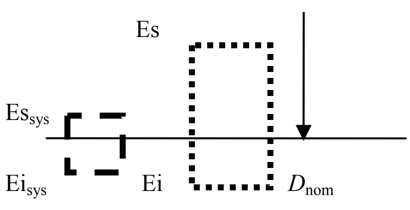

Fig 3 Dependent link tolerance (Es and Ei) and uncertainty of systematic factors (index sys)

Those values and uncertainties can include also correction factors estimation for dimensional chain links nominal parameters depending on:
- tolerance medium deviation position relating to nominal parameter,
- parameters measurements arithmetical average placement relating to nominal value in production process,
- change of parameter value through changes of temperature,
- construction detail elastic deformation;
- wearing during use,
- changes of parameter through time influence and etc.

Because X_i values are not exactly known and their values change on random, then its estimates x_i values shall be used as input.
Depending link is characterised by estimate of output Y, which can be marked as y and its values can be found through equation 1. Its values in equation 1 are N measures forming the dimensional chain and shall be used for input values $X_1, X_2,..., X_i,..., X_N$ its estimates $x_1, x_2,..., x_i,..., x_N$.
In this case estimate y is dimensional chain dependent link value and can be found using equation

$$y = f(x_1, x_2,..., x_i,..., x_N) \tag{2}$$

Estimates of input values for the dimensional chain can be presented as given in equation

$$x_i = A_i + \delta_1 A_i + \delta_2 A_i + \ldots + \delta_j A_i + \ldots + \delta_M A_i \tag{3}$$

where A_i – nominal value of dimensional chain link,
$\delta_j A_i$ – correction for the measures A_i depending on j influence factor,
M – quantity of influence factors.

Then equation (2) can be presented for min-max method of dimensional chain calculation as follows

$$A_\Delta = \\ = \sum_{i=1}^{N} \xi_i \, (A_i + \delta_1 A_i + \delta_2 A_i + \ldots + \delta_j A_i + \ldots + \delta_M A_i) \tag{4}$$

where A_Δ – depending link nominal value,
ξ_i – direction factor for dimensional chain link.

Correction factors $\delta_j A_i$ in equations (3) and (4) can be calculated in case concrete initial data is known as deviation of tolerance field TA_i for measure relating to parameters nominal value A_i or information about technological process used to assure link value is. This shall be given as arithmetical mean of size $\bar{x}_{Ai}$ relating to A_i.
Correction factors can be found also on the bases of other concrete data as correction from temperature in exploitation which reason is measures thermal expansion depending on temperature or depending on load of construction (elastic deformation).
Reason for correction is construction details' material density which depends often on humidity and resulting expansion causes the measures to change.
Depending on concrete object factors more influence can be found. All those corrections shall be taken into account for calculation value of dependent link using equation (4). Corrections may have value near to zero, but always have some uncertainty.
Using equation (4) as concrete calculation model of dimensional chain can calculate dimensional chain dependent link correction standard uncertainty depending on dimensional other links measures corrections ($\delta_j A_i$) standard uncertainties.
Usually all measures A_i of chain links are random values and dependent link A_Δ correction, caused by systematic factors, combined uncertainty can be found through summarising estimates $A_i + \delta_1 A_i + \delta_2 A_i + \ldots + \delta_j A_i + \ldots + \delta_M A_i$ combined uncertainties by equation

$$u(A_\Delta) = \sqrt{\sum_{i=1}^{N} c_i^2 u^2(A_i)} \tag{5}$$

where c_i is corrections sensitivity coefficients.

In some cases can input estimates $A_i \equiv A_i + \delta_1 A_i + \delta_2 A_i + \ldots + \delta_j A_i + \ldots + \delta_M A_i$ and $A_k \equiv A_k + \delta_1 A_k + \delta_2 A_k + \ldots + \delta_j A_k + \ldots + \delta_M A_k$ have correlation between each other then according to [3] equation

$$u(A_\Delta) = \sqrt{\sum_{i=1}^{N} c_i^2 u^2(A_i) + 2\sum_{i=1}^{N-1}\sum_{k=i+1}^{N} c_i c_k\, u(A_i, A_k)} \qquad (6)$$

is valid, where A_i and A_k are estimates for X_i and X_k and $u(A_i, A_k) = u(A_k, A_i)$ are co variations estimates of A_i and A_k.
Exceptional is case when all estimates of measures of dimensional chain have complete correlation, $r(A_i, A_k) = +1$, then equation (5) take form

$$u(A_\Delta) = \sum_{i=1}^{N} c_i u(A_i) \qquad (7)$$

Correlation factor can be calculated by equation:

$$r(A_i, A_k) = \frac{u(A_i, A_k)}{u(A_i)u(A_k)} \qquad (8)$$

where $r(A_i, A_k) = r(A_k, A_i)$,
$-1 \le r(A_i, A_k) \le +1$.

Equation (8) shows that combined uncertainty is a sum of linear values. Correlation shall be taken into account in it exists, but usually dimensional chain links are independent.

4 DIMENSIONAL CHAIN LINKS SIZE CORRECTIONS STANDARD UNCERTAINTIES

Information for standard uncertainty of correction can be found from technical figure and from details manuals.
If size is given with tolerance and assuming its normal distribution the standard uncertainty is given by equation

$$u(\delta_{\mathrm{T}} A_i) = \frac{\mathrm{T}A_i}{6} = \frac{\mathrm{Es}A_i - \mathrm{Ei}A_i}{6} \qquad (9)$$

where $\mathrm{T}A_i$ – size A_i tolerance with probability 99 %,
$\mathrm{Es}A_i$ ja $\mathrm{Ei}A_i$ – size A_i upper and lower limit deviation.

Other standard uncertainties of corrections can be expressed using equation

$$u(\delta_t A_i) = \frac{\Delta_{ti\,\max} - \Delta_{ti\,\min}}{2\sqrt{3}} \qquad (10)$$

where $\Delta_{ti\max}$ and $\Delta_{ti\min}$ are limit deviations of change of nominal size A_i for example after temperature influence. Assumed is that change distribution is according to rectangular deviation.
All standard uncertainties shall be summarised to have combined uncertainty by equation

$$u(A_i) = \sqrt{\sum_{j=1}^{M} u^2(\delta_j A_i)} \qquad (11)$$

5. CALCULATION OF TOLERANCE OF DEPENDENT LINK

Dimensional chain dependent link A_Δ correction combined uncertainty is presented as standard uncertainty.
To calculate dependent link A_Δ final limit deviations $\mathrm{Es}A_\Delta$ and $\mathrm{Ei}A_\Delta$ or measures values change interval, shall be add to dependent link tolerance correction expanded uncertainty with coverage factor $k = 3$. If systematic influence corrections cant be calculated then those correction estimates also are added to $\mathrm{Es}A_\Delta$ and $\mathrm{Ei}A_\Delta$. (Fig 3).
Coverage factor $k = 3$ gives probability level 99 % if normal distribution is under consideration which is needed for standard tolerance calculations. For specific cases other coverage factor values can be used.
In worst case corrected limit deviations of dependent link can be calculated by equation (See also Fig 4)

$$\mathrm{Es}_{\mathrm{cor}} = \mathrm{Es} + \mathrm{T}_{\mathrm{sys}}/2 \text{ and } \mathrm{Ei}_{\mathrm{cor}} = \mathrm{Ei} - \mathrm{T}_{\mathrm{sys}}/2 \qquad (12)$$

where $\mathrm{T}_{\mathrm{sys}} = \mathrm{Es}_{\mathrm{sys}} - \mathrm{Ei}_{\mathrm{sys}}$.

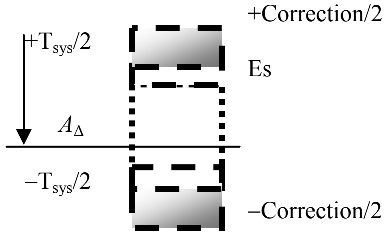

Fig 4. Dependent link summary tolerance scheme including systematic factor uncertainty and corrections if those can't be exactly estimated

Above given uncertainty estimation principles can be used to all calculations of dimensional chain.
In this case standardised tolerances of size of dimensional chain are not used but calculated and its standard uncertainties are estimated by users themselves.

6. CASE OF ILLUSTRATION FROM PRACTICE

Example: Dimensional chain has links (all with $\xi_i = +1$) with following values: A_1 = 100h6 (–0.022 mm); A_2 = (100 ± 0.15) mm; A_3 = 100H8 (+0.054 mm) and dependent link A_Δ = ?. Environmental conditions – working temperature (80±10) °C and humidity ca 80 %. Dimensional chain calculation with min-max method gives dependent link $A_\Delta = A_1 + A_2 + A_3 = 100 + 100 + 100 = 300$ mm.
Tolerance for dependent link T_Δ can be found by equation

$$\mathrm{T}_\Delta = t_\Delta \sqrt{\sum_{i=1}^{m-1} \xi_i^2 \lambda_i^2 \mathrm{T}_i^2} \qquad (13)$$

where t is risk factor, ξ is link direction factor, λ is relative standard deviation, i is tolerance unit and n quantity of links.

$\mathrm{T}_\Delta = 3\sqrt{10377.8} = 305.6$ µm ≅ 306 µm. Medium deviation of T_Δ would be $\mathrm{Ec}_\Delta = 0$ and limit deviation of dependent link are $\mathrm{Es}_\Delta = \mathrm{Ec}_\Delta + \mathrm{T}/2 = 38 + 306/2 = 191$ µm and $\mathrm{Ei}_\Delta = \mathrm{Ec}_\Delta - \mathrm{T}/2 = 38 - 306/2 = -115$ µm. This gives dependent link as $A_\Delta = 300^{+0{,}191}_{-0{,}115}$
Modified method adds corrections which take into account sizes changes through temperature and moisture influence and its uncertainty. Correction factors, in the meaning of link change depending on temperature can be calculated using equation

$$\Delta_t A_i = \alpha\,(20\ {}^\circ\mathrm{C} - t_{\mathrm{w}})\, A_i \qquad (14)$$

where α is temperature factor for linear expansion and t_{w} working temperature.

Results of calculation are given in Table 1.

Table 1

Link/ material	A_i / mm	α/ $\mathrm{m \cdot K^{-1}}$	$\Delta_t A_i$ / mm		
			min	c	max
A_1 Steel	100	$1.5 \cdot 10^{-5}$	0.075	0.090	0.105
A_2 Al-Cu-Mg	100	$2.6 \cdot 10^{-5}$	0.130	0.156	0.182
A_3 Teflon	100	$25 \cdot 10^{-5}$	1.250	1.500	1.750

Uncertainty from change of temperature ±10 °C can be estimated by assuming rectangular distribution of results, $u(\Delta t) = 10/\sqrt{3} = 5.8$ °C. Uncertainty from measurement of temperature ±1 °C assuming that rectangular distribution of results is $u(\Delta tm) = 1/\sqrt{3} = 0.6$ °C (minor importance).

Combined standard uncertainty from temperature $u(t_s) = u(\Delta t) = 5.8$ °C.

Uncertainty for linear expansion correction factor $\pm 2 \cdot 10^{-6}$ °C^{-1} (rectangular distribution) is $u(a_{A1}) = \alpha/\sqrt{3} = 2 \cdot 10^{-6}/\sqrt{3} = 1.2 \cdot 10^{-6}\ {}^0\mathrm{C}^{-1}$.

Uncertainty from linear expansion correction factor change $1 \cdot 10^{-6}$ °C^{-1} for concrete detail (link) $u(\delta\alpha) = \delta\alpha / 2\sqrt{3} = 1 \cdot 10^{-6}\,{}^0\mathrm{C}^{-1} / 2\sqrt{3} = 0.27 \cdot 10^{-6}\,{}^0\mathrm{C}^{-1}$.

Combined uncertainty for linear expansion correction factor is $u(\alpha_{1,2,3}) = 1.24 \cdot 10^{-6}$ °C.

Uncertainty for link parameter A_i is near to 0, because parameter acts in this case like constant (it is assumed to be right).

Combined uncertainty of linear size change depending on temperature and linear correction factor using equation (9) and partial derivations for first link is:

$u(\Delta_t A_i)=$

$$\sqrt{u^2(\alpha) \cdot \left(20^0\mathrm{C} - t\right)^2 \cdot A_i^2 + u^2(t) \cdot \alpha_i^2 \cdot A_i^2 + u^2(A_i) \cdot \alpha_i^2 \cdot \left(20^0\mathrm{C} - t\right)^2}$$

$$= \sqrt{(1.2 \cdot 10^{-6})^2 \cdot (20-80)^2 \cdot 100^2 + (5.8)^2 \cdot (1.5 \cdot 10^{-5})^2 \cdot 100^2 + \ldots} = \sqrt{4.8 \cdot 10^{-4}} = 0.023\ \mu\mathrm{m}.$$

Analogically calculated, results of calculations for all links are as next: $u(\Delta_t A_1) = 0.011$ µm, $u(\Delta_t A_2) = 0.017$ µm and $u(\Delta_t A_3) = 0.145$ µm.

Correction of moisture influence to link material are next (influenced is only third link): $\delta_{RH} A_3 = \Delta d_3 = 0{,}01 d_3 = 0.01 \cdot 100 = 1$ mm with standard uncertainty $u(\delta_{RH} A_3) = 0.01/2\sqrt{6} = 0.002$ mm (trapeze distribution).

Combined uncertainty for dependent link 3 from temperature and moisture influence is:

$$u(A_{\Delta 3}) = \sqrt{u^2(\Delta_t A_\Delta) + u^2(\delta_{RH} A_3)} = \sqrt{0.145^2 + 2.0^2} = 2.0\ \mu\mathrm{m}.$$

Standard summary uncertainty for links is:

$$u(\Delta_{\mathrm{SYS}}) = \sqrt{u^2(A_{\Delta 1}) + u^2(A_{\Delta 2}) + u^2(A_{\Delta 3})} = 2.0\ \mu\mathrm{m}.$$

Dependent link summary maximum movement on an average temperature is 80 °C taking account systematic factors is

$\Delta_{\mathrm{SYS}} = 0.090 + 0.156 + 1.500 + 1.000 = 2.746$ mm.

Dependent link corrected value is now

$A_{\Delta\mathrm{COR}} = A_\Delta + \Delta_{\mathrm{SY}} = 300 + 2.75 = 302.75$ mm.

Expanded uncertainty is (on 99 % probability level, $k = 3$, assuming normal distribution)

$U(\Delta_{\mathrm{SYS}}) = k \cdot u(\Delta_{\mathrm{SYS}}) = 3 \cdot 2.0 = 6.0$ µm.

Summary corrected tolerance $T_{\Delta\mathrm{COR}}$ for dependent link is

$T_{\Delta\mathrm{COR}} = T_\Delta + U(\Delta_{\mathrm{SYS}}) = 306 + 6 = 312$ µm.

CONCLUSIONS

Dependent link can be corrected by taking account systematic factors uncertainties. Systematic factors can be estimated as possible maximum and calculated by its uncertainty estimation. To get corrected dependent link size systematic factor estimate and its expanded uncertainty on probability level 99 % shall be added to the nominal value of dependent link.

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