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SUPPRESSION OF LATERAL OSCILLATIONS OF FLEXIBLE ELEMENTS IN MACHINES AND DEVICES

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Abstract: *Forced and parametric lateral oscillations of flexible elements (belts, cables, guy rapes, filaments, etc.) in machines and devices are studied. Mathematically the problem is presented as a partial differential equation describing interaction of parametric and forced vibrations of flexible element with due account of its geometrical and static non-linearities. By the mathematical simulation it is shown, that additional vibration loading of parametrically excited system can result in breakdown of parametric oscillations. On the base of this effect new approach to the suppression of unfavourable parametric vibrations of flexible elements is proposed.*
Key words: *parametric oscillations, additional excitation, flexible element, suppression.*

1. INTRODUCTION

Flexible elements (belts, cables, guy ropes, filaments, strings, etc.) are widely used in machines and devices for various practical purposes (belt and chain transmissions, vibrating belts of vibromixers, etc). Lateral vibrations of flexible elements, which can occur during the operation of machine, are extremely detrimental (Armada et al., 2003). They give rise to additional dynamic loading, which encourages wearing and failure of flexible elements.

Spectrum of resonance lateral oscillations of flexible elements may be sufficiently dense (resonances of forced vibrations, simple parametric resonances, combination resonances). Besides, geometrical and physical non-linearity of flexible element can result in pulling of resonant oscillations and further widening of dangerous frequency ranges. In such conditions system's tuning away from resonance frequencies remains problematic.

This paper proposes new approach to the suppression of non-linear parametric vibrations based on application to the system of additional kinematical vibration excitation. Effect of vibration stabilization is known in mechanics (Landa, 1996; Oks et al., 1993; Strizhak, 1984). Typical example is the stability of overturn pendulum in the case of vibration action on hanging point (Chelomej, 1956). The present research is devoted to studying of vibration stabilization phenomena in application to a parametrically excited flexible element (thread).

2. DYNAMIC MODEL

Transverse oscillations of taut flexible element (thread) under parametric and kinematical excitations are considered (Fig. 1). Parametric excitation is caused by periodic variation in time of axial tension force, but kinematical excitation is due to forced transverse displacement of one end of the flexible element.

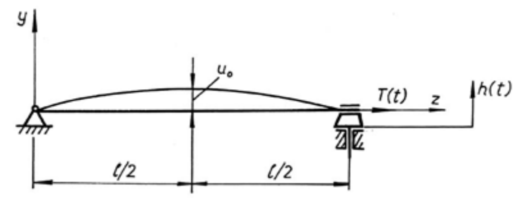

Fig. 1. Model considered in dynamic analysis

In forming of differential equation of oscillations some assumptions are made. It is supposed, that stiffness in bending of flexible element is negligible in comparison with its stiffness in tension, but weight of flexible element is ignorable in comparison with axial prestressing force T_0. Besides, it is considered that oscillations are performed in one plane, which runs along the centre line of a non-deformed flexible element. Taking the direction of the co-ordinate axis z along this centre line, the differential equation for transverse vibrations of flexible element can be stated as follows (Bondar, 1971; Tsyfansky et al., 1991):

$$T_0(1+\mu\sin\Omega t)[1+f(\varepsilon)](1+b_1\frac{\partial}{\partial t})\frac{\partial^2 y}{\partial z^2}-b_2\frac{\partial y}{\partial t}-\rho[1+\frac{1}{2}(\frac{\partial y}{\partial z})^2]\frac{\partial^2 y}{\partial t^2}=0\,, \tag{1}$$

where T_0 is the prestressing force of flexible element; μ and Ω are the non-dimensional amplitude and the frequency of parametric excitation; b_1 and b_2 are the coefficients of internal and external friction; y is the lateral displacement of the flexible element's cross-section with the co-ordinate z.

The functional $f(\varepsilon)$ in equation (1) takes into account additional tension caused by elastic deformation of flexible element during its oscillations (physical non-linearity). The elongation ε of flexible element can be determined by formula (Bondar, 1971):

$$\varepsilon=\frac{1}{2l}\int_0^l(\frac{\partial y}{\partial z})^2dz\,, \tag{2}$$

where l is the length of flexible element.

The relationship between axial stress σ in flexible element and its elongation ε can be approximately described by the expression

$$\sigma=E\varepsilon-\beta\varepsilon^3\,, \tag{3}$$

where E is the elasticity modulus of material; β is the coefficient of non-linearity. In this case the functional $f(\varepsilon)$ can be expressed in the following form

$$f(\varepsilon)=\frac{EA}{2T_0 l}\int_0^l(\frac{\partial y}{\partial z})^2dz-\frac{\beta A}{8T_0 l^3}[\int_0^l(\frac{\partial y}{\partial z})^2dz]^3\,, \tag{4}$$

where A is the cross-section area of flexible element.

Therefore an increment in tension is caused by integral elongation of flexible element and is independent of co-ordinate z. Non-linear term $[1+\frac{1}{2}(\frac{\partial y}{\partial z})^2]$ of equation (3) takes into account geometrical non-linearity of flexible element (Bondar, 1971).

In the case studied here the end boundary conditions are as follows:

$$y(z=0,t)=0\,; \qquad y(z=l,t)=h\sin\omega t\,, \tag{5}$$

where h and ω are the amplitude and frequency of external kinematical excitation.

Equation (1), subject to the expressions (4) – (7), was solved on an analogue-digital computer system predominantly set up for the solution of complex non-linear dynamics problems (Tsyfansky et al., 1991). The integration of non-linear differential equations is carried out on the high-speed analogue part of the computer system, but control over the programming of the analogue part and data processing is executed by the digital part. The methods of mathematical simulation and the operational principle of the computer system are described in more detail in references (Tsyfansky, 1979; Tsyfansky et al., 1991). The quantitative estimation of accuracy in analogue-digital simulation was carried out by the solution of test examples and particular engineering problems (Belovodsky et al., 2002; Tsyfansky et al., 1991; Tsyfansky & Beresnevich, 2000).

3. PARAMETRIC VIBRATIONS OF FLEXIBLE ELEMENT

First the case of parametric excitation only (h = 0) is considered. Fig. 2 shows the region of main parametric resonance ($\Omega = 2\omega_1$, where ω_1 is the first natural frequency of lateral oscillations of flexible element) in the plane of parameters μ and $\eta = \Omega/\omega_1$. Above a typical amplitude-frequency characteristic (AFC) corresponding to the first mode of transverse oscillations of flexible element is presented (resonance curve *abc*). This graph is constructed assuming the parameters of the equations (1) – (5) to be according to the following conditions: $\rho lg/EA = 6\cdot10^{-6}$; $b_1\omega_1 = 0.003$; $T_0/EA = 2\cdot10^{-4}$; $\mu = 0.15$. Dimensionless displacements u_0/l are projected as amplitudes on this AFC.

Part *ab* of the resonant curve corresponds to the parametric regimes realized with the system tuning on the main region of parametric instability. The domains of attraction for this case (η = 2) are shown in Fig. 3. The origin of co-ordinates ($y_{(z=l/2,\ 0)} = 0$, $\dot{y}_{(z=l/2,0)}=0$) is a saddle point (trivial solution, corresponding to the unstable rest state of the system). Two stable focal points $S'_{1/2}$ and $S''_{1/2}$ correspond to two stable parametric regimes, which are in antiphase and equal in amplitudes. Either parametric regime has its own domain of attraction.

Part *bc* of the AFC corresponds to the zone of non-linear pulling of vibrations. In this case amplitudes of oscillations are sufficiently increased. Fig. 4 shows the domains of attraction of parametric regimes for the case η = 2.7 and $z = l/2$. There are two domains of initial conditions, which lead the system to stable parametric regimes (stable focal points $S'_{1/2}$ and $S''_{1/2}$. All other initial conditions drive the system to the stable trivial solution $y(z,\ t)$ = 0 and $\dot{y}(z,t)=0$ (stable focal point S_0).

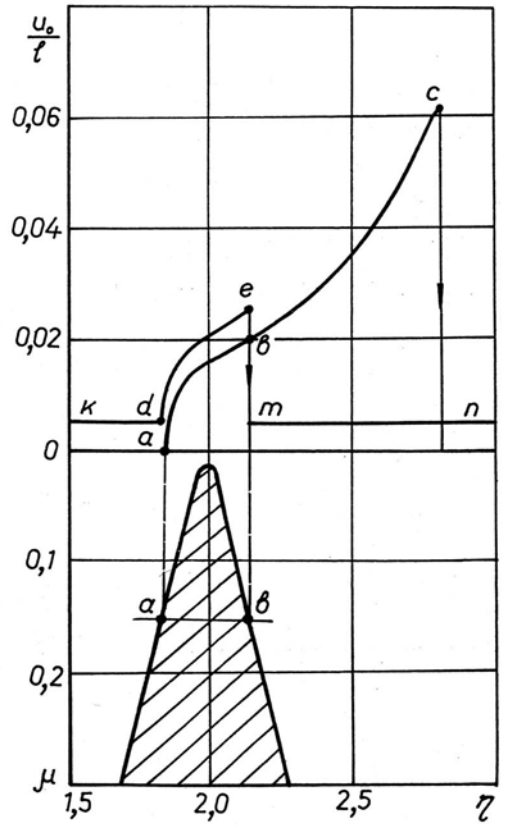

Fig. 2. Region of main parametric resonance and typical amplitude-frequency characteristics

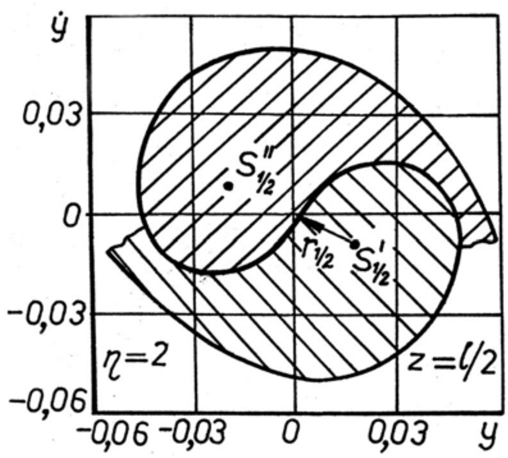

Fig. 3. Domains of attraction for the regimes realized inside the main region of parametric instability (η = 2)

As follows from the analysis of the domains of attractions presented, parametric oscillations of flexible element have a limited reserve of stability. For example, the loss of stability is possible under some perturbation of phase co-ordinates y, $\dot{y}$ of parametric regime. If one of two regimes (e.g., regime $S'_{1/2}$) loses stability inside region of parametric resonance (part *ab* of the AFC), then instead of regime $S'_{1/2}$ other regime $S''_{1/2}$ is excited (see Fig. 3). Amplitude and frequency of parametric oscillations in this case are unchanged.

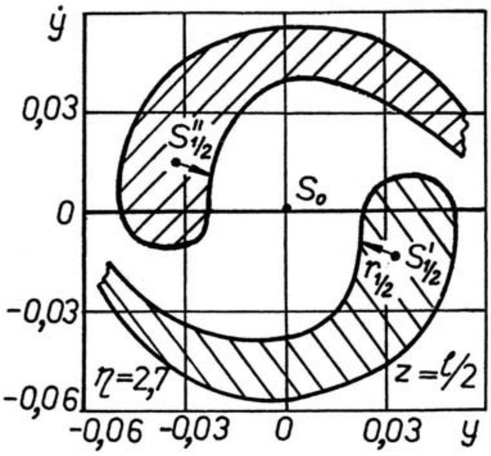

Fig. 4. Domains of attraction for the regimes realized outside the main region of parametric instability (η = 2.7)

Disturbance of phase co-ordinates $y(z, t)$ and $\dot{y}(z,t)$, if it takes place inside the pulling zone *bc*, can result in breaking down the steady state parametric oscillations. In this case system reaches the quiescent state (stable focal point S_0) instead of oscillatory regimes $S'_{1/2}$ and $S''_{1/2}$ with finite amplitude (see Fig. 4).

4. INTERACTION OF PARAMETRIC AND FORCED VIBRATIONS OF FLEXIBLE ELEMENT

As it follows from the analysis of solutions of the set of equations (1) – (5), the disturbance of phase co-ordinates $y(z, t)$ and $\dot{y}(z,t)$, which drives the system to the breaking down of non-linear parametric oscillations within frequency range *bc* (Fig. 2), can be realized by the proper choice of parameters ω and h of the external kinematical excitation. What is more, under certain values of parameters ω and h it is possible to suppress completely the parametric oscillations in the pulling zone *bc*.

Values of ω and h, which favour such suppression, have been determined by the mathematical simulation of the problem stated on the base of factorial experiment design. During this simulation main dimensionless parameters of the system have been varied within the limits: $T/EA = (0.5 \div 5)\cdot 10^{-4}$; $b_1\omega_1 = 0.002 \div 0.018$; $\eta = \Omega/\omega_1 = 2 \div 3$; $\mu = 0 \div 0.5$. Experimental points have been located within this space of parameters in accordance with the uniform distribution design (Audze & Eglais, 1977).

It is shown by the mathematical simulation, that application to flexible element of external kinematical excitation $h\sin\omega t$ is effective in the low frequency range ($\nu = \omega/\omega_1 = 0.2 \div 0.8$). In this case suppression of non-linear parametric oscillations is achieved under the least possible amplitude $h = (0.25 \div 0.33)u_0$ of kinematical excitation. The non-linear effect revealed is used for the development of new method for the suppression of parametric oscillations of flexible element.

Fig. 2 shows a resonance curve *de*, which corresponds to the resulting oscillations of flexible element after application to it of kinematical excitation $h\sin\omega t$ with parameters $h = 0.25u_0$; $u_0 = 0.05l$ and $\nu = 0.5$.

It follows from the comparison of curves *de* and *abc* that due to application to the system of additional kinematical excitation the non-linear pulling of parametric oscillations is completely avoided. And simultaneously some amplification of the resulting oscillations in region *ab* (approximately on 25%) is observed. Besides, outside the frequency range *ab* usual forced oscillations with frequency ω are excited (parts *kd* and *mn* of the AFC). But on the whole, the intensity of transverse vibrations of flexible element after application of additional kinematical excitation is sufficiently decreased.

5. CONCLUSION

New approach to the suppression of unfavourable non-linear parametric oscillations of flexible elements (belts, cables, guy ropes, etc.) based on application to the system of additional vibration excitation is proposed. This method makes it possible to prevent non-linear pulling of resonant oscillations and thanks to this extends the allowable operating frequency range of machine. This is especially important in cases, when tuning away from hazard resonant frequencies is hindered or impossible at all. Structural realization of the method proposed in many cases is simplified, because vibration of machine itself can be used as external kinematical excitation.

6. REFERENCES

Armada, M.; Gonzales de Santos, P. & Tachi, S. (2003). *Measurement and Control in Robotics*, Instituto de Automatica Industrial, ISBN: 84 – 607 – 9693 – 0, Madrid.

Audze, P. & Eglais, E. (1977). New approach to the design of multi-factor factorial experiments, In: *The Problems of Dynamics and Strength*, Lavendelis, E. (Ed.), No. 35, p. 92 – 103, Publ. House Zinatne, Riga. (In Russian).

Belovodsky, V.; Tsyfansky, S. & Beresnevich, V. (2002). The dynamics of a vibromachine with parametric excitation. *Journal of Sound and Vibration*, Vol. 254, No. 5, p. 897–910, ISSN: 0022-460X.

Bondar, N. (1971). *Nonlinear Autonomous Problems in Mechanics of Elastic Systems*, Budivelnik, Kiev. (In Russian).

Chelomej, V. (1956). On the possibility to increase the stability of elastic systems with the aid of vibration. *Reports of the Academy of Sciences of the USSR*, Vol. 110, No. 3, p. 345 – 347. (In Russian).

Landa, P. (1996). *Nonlinear Oscillations and Waves in Dynamical Systems*, Kluwer Academic Publisher, ISBN: 0792339312, Dordrecht / Boston / London.

Oks, A.; Yano, S.; Tsyfansky, S. & Iwatsubo, T. (1993). Suppression phenomena of resonant oscillations in strongly nonlinear systems due to additional asynchronous excitations. *JSME International Journal*, Series C, Vol. 36, No. 1, p. 45 – 51.

Strizhak, T. (1984) *Asymptotic Method of Normalization: Averaging Method and Method of Normal Forms*, Vishcha Shkola, Kiev. (In Russian).

Tsyfansky, S. (1979). *Electric Simulation of Oscillations in Complex Nonlinear Mechanical Systems*, Zinatne, Riga. (In Russian).

Tsyfansky, S.; Beresnevich, V. & Oks, A. (1991). *Nonlinear and Parametric Oscillations of Technological Vibration Machines*, Zinatne, ISBN: 5-7966-0153-9, Riga. (In Russian).

Tsyfansky, S. & Beresnevich, V. (2000). Non-linear vibration method for detection of fatigue cracks in aircraft wings. *Journal of Sound and Vibration*, Vol. 236, No. 1, p. 49 – 60, ISSN: 0022-460X.

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