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EXPERIMENTAL AND THEORETICAL RESEARCH OF STIFFNESS OF BONDED HOLLOW CYLINDRICAL RUBBER BLOCK

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¶ **Abstract:** *Problem of axial loading of axisymmetrical elastomeric compensative device is discussed. We consider behaviour of a rubber hollow circular cylinder with flat ends bonded to perfectly rigid plates.*
Analytical expression for compressive stiffness was derived based on a principle of minimum of additional potential energy.
Numerical results, received according to analytical expression, coincide with the experimental data. The experiments were performed on Zwick/Roell Z-150 machine equipment.

Key words: rubber, compression stiffness, Young's modulus, isolator, Ritz' method.

1. INTRODUCTION

Rubber-like materials (elastomers) exhibit specific properties: high elasticity, good dynamic properties, low volume compressibility (Poisson's ratio μ = 0.480 - 0.495), a linear relationship between stress and strain up to strain of 15% ÷ 20%, resistance to environmental factors. Due to these properties rubber-like materials are used as elastic members in various types of vibration isolators and compensation devices: elastic gaskets, cushions, shock absorbers, dynamic vibration absorbers, resonance and antiwaveguide systems, dampers, buffers, stops, etc. Despite the diversity of construction designs, these elements are generally reduced to the calculation model of rubber elements: rubber cylinder of finite length with a stress-free side surface of various types of boundary conditions on the ends; hollow rubber cylinder and cylindrical shell of finite length. Rubber blocks, consisting of rubber elements bonded between two rigid plates, are widely used in many engineering applications. Operating mode of elastic cylinders depends on many factors that must be considered at the design stage: parameters of the elements must be selected properly. The most important parameter is compression stiffness – analytical dependence between imposed force and received deformation.
In this study the work of axially symmetrical elastic hollow circular cylinder, reinforced at the flat ends by rigid plates, under axial compressive loading is considered (Fig. 1).
Many researchers study the behaviour of elastic device [1-5]. Most of these studies deal with the axial compression of the rigidly-bonded rubber layer and their results allowed to establish basic assumptions for small deformation and linear analysis: horizontal plane sections remain plane and initially vertical lateral surfaces take a parabolic shape after deformation, state of stress at any point in the material is dominated by hydrostatic pressure. Since precise analytical methods for solving often pose certain problems, an approximate analytical and numerical methods, based on variational principles, are of considerable interest.
In this work to determine the compressive stiffness of the rubber blocks variotional Ritz method are applied, using a principle of minimum of additional potential energy. To determine the stiffness at small deformations the linear theory of elasticity is used. The objectives of given work are to develop the analytical force - displacement

dependence for hollow rubber cylinder, to fulfil the relative experiments, to compare the theoretical results with experimental data. The experiments were performed on HB Zwick Roell Z-150 loading machine.

2. ANALYTICAL MODELS OF BONDED RUBBER BLOCK

Designed object - axysimmetrical rubber hollow cylinder of finite length with a stress-free inner and outer side surfaces and plane ends bonded to rigid plates under axial compression is presented in Fig.1.

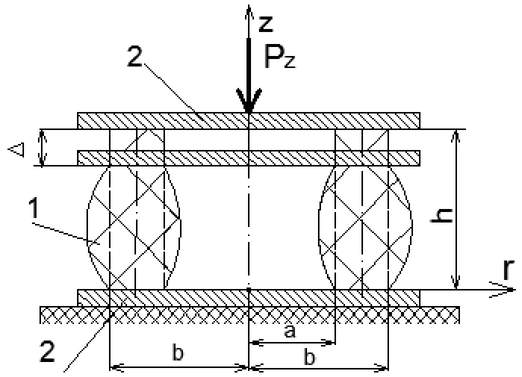

Fig. 1. Scheme of research object: 1-rubber element, 2 - rigid plate; a, b - inner and outer diameters, h – height, Δ -deformation

For axially symmetric problem cylindrical coordinates system *r, φ,* z are chosen.
Solution is performed for small deformation (~10÷15%). taking into account elastomeric weak volume compressibility (Poisson ratio μ < 0.5). End plates are assumed perfectly rigid, plane horizontal sections of rubber are considered plane after deformation. The dependence of the force –displacement *"P – Δ"* are defined by Ritz's method, minimizing the additional potential energy of deformation with selected of stress functions as required functions [6] [7]:

$$\Pi = \iiint_V W(\sigma_{ij})dV - \int_{F_u} \sigma_{ij} n_j u_i dF \quad (1)$$

$$W(\sigma_{ij}) = \frac{\sigma^2}{2K} + \frac{\sigma_\tau{}^2}{6G},\ K = G\frac{2(1+\mu)}{1-2\mu},$$

$$\sigma = \frac{1}{3}(\sigma_{rr} + \sigma_{\varphi\varphi} + \sigma_{zz}),$$

$$\sigma_\tau = \frac{1}{2}\begin{bmatrix} (\sigma_{rr} - \sigma_{\varphi\varphi})^2 + (\sigma_{\varphi\varphi} - \sigma_{zz})^2 + \\ + (\sigma_{zz} - \sigma_{rr})^2 + 6\sigma_{r\varphi}^2 \end{bmatrix},$$

where: K – bulk modulus of elastomer, G – shear modulus, μ – Poisson ratio, σ – mean stress (called spesific hydrostatic pressure).
Stress state of the elastomer is determined by the superposition of shear stress on the hydrostatic pressure. Stress components in equations (1) must satisfy the equilibrium equations and boundary conditions on the surface of the elastomer cylinder. For small axial strain (*Δ/h* << 1) considering axial symmetry, we accept

$$\sigma_{rr} \approx \sigma_{\varphi\varphi} = s(r),\ \sigma_{zz} = \sigma_{zz}(r),$$

$$\sigma_{r\varphi} = \sigma_{\varphi z} = 0,\ \sigma_{rz} = -z\frac{\partial s}{\partial r}. \quad (2)$$

The relationships (2) satisfy equilibrium equations in a volume and must satisfy stresses boundary conditions:

$$s_{rr}(r = a,b) = s(a,b) = 0 \quad (3)$$

$$\int_{-0.5h}^{0.5h} \sigma_{rz}\Big|_{r=a,b} dz = \int_{-0.5h}^{0.5h} z\frac{ds}{dr}\Big|_{r=a,b} dz \equiv 0$$

Accounting (2) and (3) functional (1) is written:

$$\Pi = \frac{2\pi h}{6G}\int_a^b \begin{bmatrix} \frac{G}{3K}(\sigma_{zz} + 2s)^2 + (\sigma_{zz} - s)^2 + \\ + \frac{h^2}{4}\left(\frac{ds}{dr}\right)^2 + 6G\frac{\Delta}{h}\sigma_{zz} \end{bmatrix} rdr \quad (4)$$

In view of symmetry of problem and conditions (2), (3) stress functions are selected:

$$\sigma_{zz} = C_1 s(r) + C_2,$$

$$s(r) = C_3\left(1 - \frac{r^2}{b^2} - \frac{1-\alpha^2}{\ln(\alpha)}\ln\left(\frac{r}{b}\right)\right). \quad (5)$$

From the condition of minimization of functional (4)

$$\frac{\partial \Pi(C_1, C_2, C_3)}{\partial(C_1, C_2, C_3)} = 0$$

constants C_1, C_2, C_3 are found:

$$C_1 = \frac{1 - \frac{2}{3}\frac{G}{K}}{1 + \frac{1}{3}\frac{G}{K}}, \quad C_2 = -\frac{3G\Delta}{h\left(1 + \frac{1}{3}\frac{G}{K}\right)}, \quad (6)$$

$$C_3 = -3G\frac{\Delta}{h}\frac{b^2}{h^2}\left(1 - \frac{r^2}{b^2} - \frac{1-\alpha^2}{\ln(\alpha)}\ln\left(\frac{r}{b}\right)\right).$$

Force - displacement dependence "P_z - Δ" is found from the condition of equilibrium on the middle surface of rubber block:

$$P_z = -2\pi\int_a^b \sigma_{zz} r dr\,. \qquad (7)$$

From the Equations (5) – (7) we receive:

$$P = \frac{9AG\Delta}{h(3+\nu)}\left[1 + \frac{\rho^2(3-2\nu)^2\left(1+\alpha^2 + \frac{1-\alpha^2}{\ln\alpha}\right)}{6(3+\nu)\left(1 + \frac{9\rho^2\nu}{3+\nu}\frac{B_1}{B_2}\right)}\right] \qquad (8)$$

$$B_1 = \frac{2(1-\alpha^6)}{3} - \frac{(1-\alpha^2)^3}{\ln\alpha} + \frac{3(1-\alpha^4)(1-\alpha^2)}{2\ln\alpha}$$

$$B_2 = (1-\alpha^2)\left(1+\alpha^2 + \frac{1-\alpha^2}{\ln\alpha}\right)$$

$$\nu = \frac{G}{K} = \frac{1-2\mu}{2(1+\mu)},$$

$$\rho = b/h, \quad \alpha = a/b, \quad A = \pi\left(b^2 - a^2\right).$$

For uncompressible material ($\mu=0.5$):

$$P = \frac{3AG\Delta}{h}\left[1 + 0.5\rho^2\left(1+\alpha^2 + \frac{1-\alpha^2}{\ln\alpha}\right)\right]$$

considering $3G=E$ (E – Young's modulus) we have:

$$\Delta = \frac{Ph}{AE}\left[1 + 0.5\rho^2\left(1+\alpha^2 + \frac{1-\alpha^2}{\ln\alpha}\right)\right]^{-1}, \quad (9)$$

or $\Delta = \frac{Ph}{AE_a}$, where $E_a = E\beta$,

$$\beta = 1 + 0.5\rho^2\left(1+\alpha^2 + \frac{1-\alpha^2}{\ln(\alpha)}\right).$$

E_a is called apparent Young's modulus.

Based on previous researches Gent et al. developed the approach to stiffness of bonded rubber block definition, named "pressure method". Gent's ″pressure method″ is widely used approach since his formulation usually leads to relatively simple expressions. Gent et al. based on "pressure method". showed that force-displacement characteristics under compressive loading depends on Young's modulus E and shape factor s, defined as the ratio of one loaded area to the total stress free area [8] [9]:

$$E_a = E(1 + 2s^2),$$

where s – shape factor: using agreed in this paper notation

$$s = \frac{A_{load}}{A_{free}} = \frac{b-a}{2h} = \frac{\rho}{2}(1-\alpha) \qquad (10)$$

$$\Delta = \frac{Ph}{AE(1+2s^2)} = \frac{Ph}{AE\left(1 + 0.5\rho^2(1-\alpha)^2\right)}$$

For solid rubber cylinder (a=0 and α=0) the equations (9) and (10) coincide. Gent's approach does not allow to take into account elastomers weak compressibility. If displacement of rubber block is too large the stiffness may be increased by adding the rigid plate and decreasing the height of rubber elements as it shown in Fig.2. Then the total displacement will be equal to the sum of blocks displacement and stiffness – one block stiffness divided on the number of blocks.

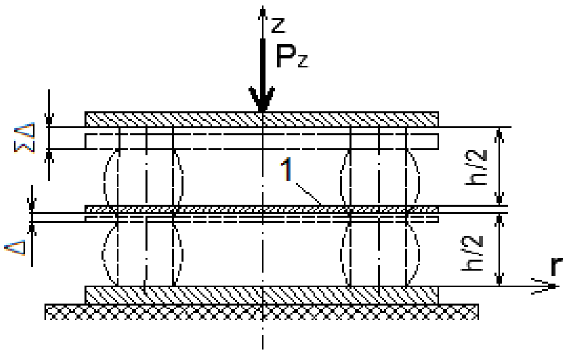

Fig. 2. Block of two interlocked rubber elements bonded to rigid plate 1.

In accordance with (9) for the case shown in Fig. 2 total displacement is equal:

$$\Delta = \frac{Ph}{AE}\left[1 + 2\rho^2\left(1 + \alpha^2 + \frac{1-\alpha^2}{\ln\alpha}\right)\right]^{-1}.$$

3. RESULTS OF EXPERIMENTS

Rubber blocks with dimensions: a=40 mm, b=50mm, h=18 mm, mechanical properties G=2.6 MPa, μ=0.495, presented in Fig. 3, were used for preparation of testing specimens. Three types of specimens were manufactured: one bonded rubber block, two and three bonded rubber blocks, presented in Fig.4. All specimens were tested under different mode of loading: Experiments was carried out with the number of specimens on Zwick/Roell Z-150 machine equipment (Fig. 5).

Fig. 3. Rubber block of testing specimen

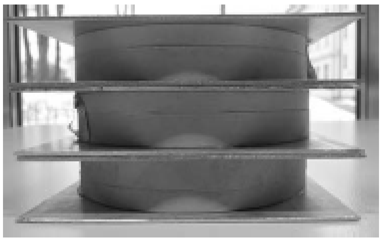

Fig. 4. Reinforced testing specimen: block with 3 rigid bonded rubber hollow cylinder

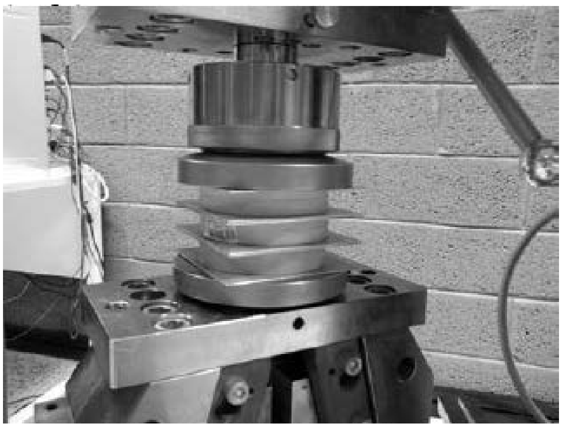

Fig. 5. Testing machine with specimen

Plots of force – displacement "P – Δ" dependence graphed according to the analytical solution for one bonded rubber block are presented in Fig. 6, for three bonded rubber blocks - in Fig. 7.

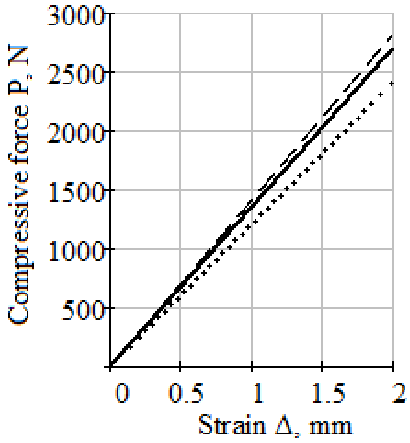

Fig. 6. Plots of "P – Δ" dependence for one bonded rubber block with accordance to formulas: ···· (8); ―― (9); — -(10)

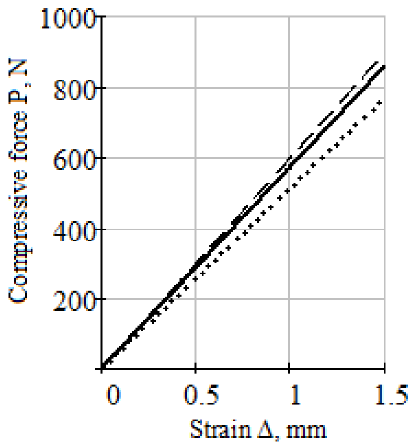

Fig. 7. Plots of "P – Δ" dependence for specimen with three bonded rubber blocks with accordance to formulas: ···· (8); ―― (9); — - (10)

Loading - unloading of specimens was performed with different speed. Speed of loading equal 0.167, 0.333, 0.500, 0.667 mm/s show the constant apparent Young's modulus, this loading may be considered as static. With the increasing of speed the apparent Young's modulus increases. Loading cycles number were 2,5,10,20, 50. The results of testing of one bonded rubber block are presented in Fig. 8-11, package of two bonded blocks – in Fig. 12 and three bonded rubber block - in Fig. 13.

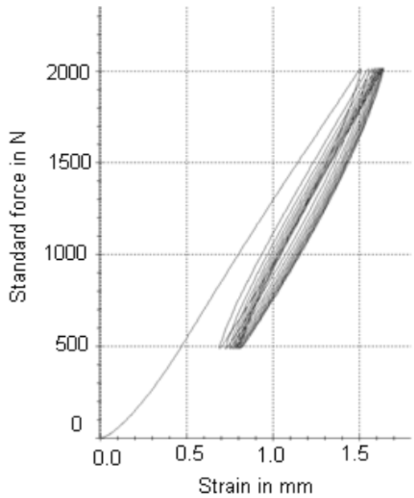

Fig. 8. Results of compression testing of one block specimen with the speed of loading –unloading 0.167 mm/s, 10 cycle

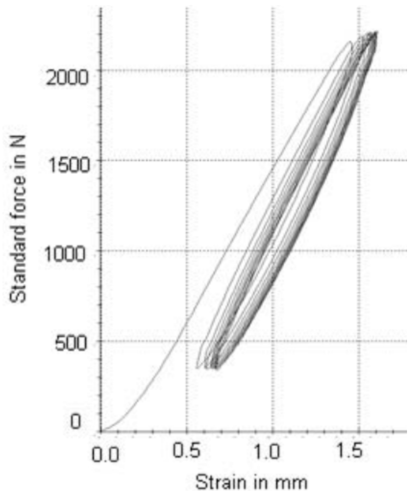

Fig. 9. Results of compression testing of one block specimen with the speed of loading 1.667 mm/s, 10 cycles

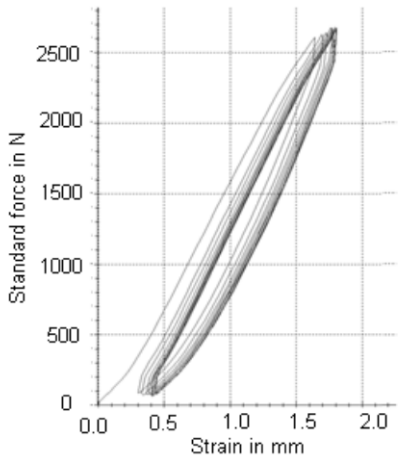

Fig. 10. Results of compression testing of one block specimen with the speed of loading 5.00 mm/s, 10 cycles

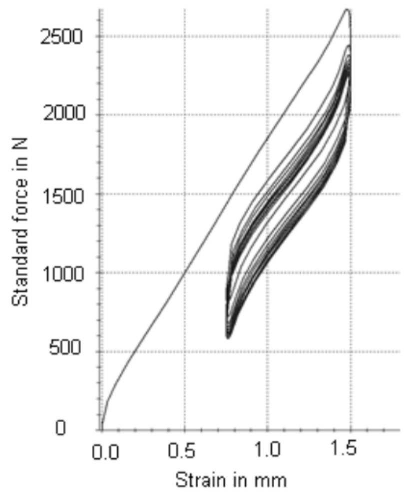

Fig. 11. Results of compression testing of one block specimen with the speed of loading -unloading 15 mm/s, 10 cycles

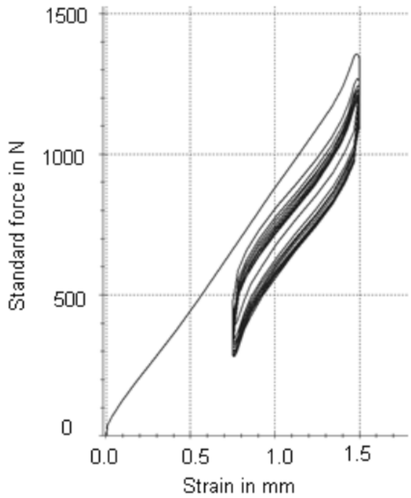

Fig. 12. Results of compression testing of two blocks specimen with the speed of loading - unloading 15 mm/s, 10 cycles

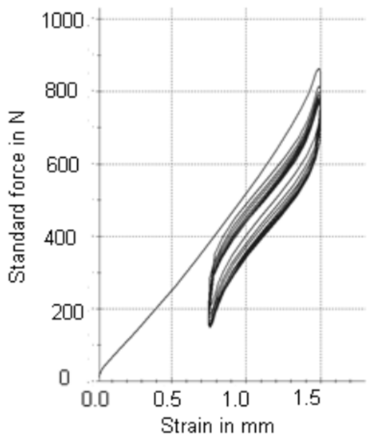

Fig. 13. Results of compression testing of three blocks specimen with speed of loading- unloading 15 mm/s, 10 cycles

4. CONCLUSION

• The behaviour of rubber hollow circular cylinder, rigidly bonded to perfectly rigid plates, under axial compressive load is considered.

•Analytical expression for compressive stiffness as ″P – Δ″ dependence was derived using variational method with Ritz procedure, based on principle of minimum of additional potential energy.

• Numerical results, received in accordance with analytical expression, show good coincidence with the experimental data in the case of static loading.

• Experiments in compression of bonded hollow rubber block was carried out on the number of specimens on Zwick/Roell machine equipment. Loading - unloading of specimens was performed with different speed. Speed of loading equal 0.167, 0.333, 0.50, 0.667 mm/s show the constant apparent Young’s modulus. This type of loading may be considered as static loading. With the increasing of speed the apparent Young’s modulus increases.

• Apparent Young’s modulus dependence on loading speed is the topic of author’s future investigations.

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