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MODEL OF PRECISION PARTS ASSEMBLY

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Abstract: The paper presents a model of precision parts assembly. The model is applicable for solving problems of creating systems of analysis and synthesis of product tolerances. The model is based on simulation of mating parts surfaces. To simulate mating parts an upgraded algorithm of the nearest points (ICP) is used. The model was used for the numerical simulation of assembly of parts that are joined by cylinder and plane surfaces. The developed model can be used to improve the accuracy of manufacturing processes, and repair of complex industrial products.
Key words: uncertainty evaluation, manufacturing, assembly, tolerance synthesis.

1. INTRODUCTION

Key Product Characteristics (KPCs) is largely dependent on the geometrical accuracy of parts and assembly units. Conjugation of surfaces of machine parts forming the assembly dimensional chains, which include master links. A dimensional chain consists of separate partial parts (input dimensions) and ends with a closed part (resulting dimension). For example, in the turbomachinery resultant sizes may be radial and face runouts of rotor surfaces and clearance formed between the tips of the blades and the stator. Dimensional chains are formed by mating surfaces of adjacent components. During manufacture of parts the size errors are formed affecting the mutual position of the surfaces and the error of surface shapes. When calculating dimension chains two problems such as Tolerance analysis - direct tasks, control; Tolerance synthesis - indirect tasks, designing can be solved. To solve dimension chains we can use the following methods: an arithmetic method of calculation, a statistical method of calculation, a method of group interchangeability. Mentioned methods do not accurately account for the surfaces form error of parts in the calculation of dimension chains. For precise metering inaccuracies of form and position of parts surfaces it is necessary to develop a method that takes into account imperfect mating surfaces of parts. The basis of the method will be a model of precision parts assembly that allows defining attainable relative position of mating parts. This paper describes a model of precision parts assembly. Many sources [1-4] are devoted to aircraft engine manufacturing and assembly technologies.

2. PROPOSED METHOD

Formation of dimensional relationships in the process of manufacture and assembly of machines is largely determined by surface conjugation. The mating of surfaces is complex process and it has a probabilistic uncertainty, which is defined by the assembly conditions (manufacturing procedure) and geometrical deviations. The uncertainty of the mating surfaces involves variation of relative positions of the assembled parts surfaces. In general, the relative position of the parts is described by six components. Three components define a linear arrangement. The other three components determine the angular placement. The assembly conditions include a fastening technique for assembling parts, its orientation in space and the direction of the application vector of assembly force.

The paper deals with the assembly model of mating surfaces, which are considered as absolutely rigid bodies. Such an assumption is justified when relatively rigid parts are assembling as well as in the case of the stepwise computation taking into account the deformation of parts and their surfaces under the influence of assembly effort. Determination of the uncertainty of the mating surfaces of parts is performed by varying assembly conditions and the relative positions of the assembling parts.

We consider the creation of a model of the assembly process by the example of mating two parts with cylindrical and flat surfaces. This conjugation is very common in the structure of different mechanisms and machines.

2.1 Mathematical model for mating surfaces

The model of conjugation of curves or surfaces is based on the method of best fit with set of constraints on the intersection of surfaces. The conjugation of two surfaces with form deviations can be characterized by the size of the clearance G between the surfaces. In the case of the intersection of the surfaces the negative clearance is formed that is interference. In general terms, the clearance / interference can be represented as a spatial function, every point of which is determined by the difference of point's coordinates of the mating surfaces. The spatial function of clearance characterizes the achievement of co-state for parts surfaces and depends on the $\overline{V}$ vector of relative surfaces position. In general the relative position vector includes three linear and three angular parameters for the respective axes. Let us set the clearance function is $G(\overline{V})$.

We developed an iterative algorithm to calculate the conjugation of parts. The co-state algorithm assumed iterative movement of mating surface relative to another one with the $\overline{D_1}$ application vector of assembly effort. To calculate the function $G(\overline{V})$ the best fit of surfaces is performed at each stage. A common algorithm for solving the problem of the best fit is an iterative algorithm of the nearest points (ICP) presented in [5]. According to this algorithm at the each iteration of the search the rotation angles and displacements along the coordinate axes are calculated by methods of nonlinear optimization. Taking into account the assembly force application vector the objective function of algorithm can be defined as follows:

$$f(R,t)=\frac{1}{n_p}\sum_{i=1}^{N} w_i \left\| R\cdot \overline{p}_{surf2\_i}+\overline{T}-\overline{p}_{surf1\_i}\right\|^2 \rightarrow 0, \quad (1)$$

where n_p is the number of matching points;

$\overline{p}_{surf1\_i}$ is the vector of the first surface coordinates;

$\overline{T}$ is the displacement vector (contains 3 displacements along the coordinate axes);

R is the rotation matrix (contains 3 angles of rotation around the coordinate axes);

$\overline{p}_{surf2\_i}$ is the vector of the second surface coordinates corresponding to $\overline{p}_{surf1\_i}$;

w_i is the weighting factor for the distance between the points of the surfaces.

The weighting factor for each point is calculated as follows:

$$w_i=\delta-\left\|\overline{D}_1-\overline{n}_i\right\|, \quad (2)$$

where δ is constant which equals to the maximum value of the difference between assembly force application vector and normal vector at the point of the mating surface;

$\overline{n}_i$ are the normal vectors of the ith point of the first mating surface.

To search for function parameters (1) we apply the method of sequential quadratic programming, which is one of the methods of nonlinear optimization To eliminate intersection of two surfaces we use a inequality system presented in [6], it apply restrictions on the clearance function $G(\overline{V})$:

$$\begin{cases}(\overline{p}_{пов2\_1}-\overline{p}_{пов1\_1})^T\cdot\overline{n}_1\geq 0,\\ \quad\quad\ldots\\ (\overline{p}_{пов2\_i}-\overline{p}_{пов1\_i})^T\cdot\overline{n}_i\geq 0;\end{cases} \quad (3)$$

The algorithm takes into account the mating conditions, which are formalized by the motion vector $\overline{D_1}$ in view of surfaces crossing terms.

2.2 Model of mating surfaces

The mating surfaces have a complex shape due to the deviations [7]. Curves and surfaces of complex shape are described by spline equations in CAD-systems and in measurement assurance of measuring apparatus. The spline is piecewise polynomial to the power K with a continuous derivative to the power K-1 at the points of segments connection called setting points. For the mathematical representations of complex surfaces we used normalized cubic spline to the power 3 also known as Hermitian curve and described in [8].

To describe surfaces of parts with geometric deviation form we use surface formed by bicubic patches (Coons patches). Described surface represents a segment corresponding to the parameters values $0 \le u \le 1$, $0 \le v \le 1$. The Coons patch is formed by the conjunction of boundary spline curves and it is defined by:

$$P(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} u^i v^j \ , \tag{4}$$

where a_{ij} is an algebraic vector coefficients with constituents x, y and z.

Combining of Koons patches makes possible to determine the surface of any shape and size. The spline surface is defined in the parameter space u and v.

3. RESULTS

The paper presents the connection of the shaft flange and the disc including flat and cylindrical surfaces, which are shown in Fig. 1.

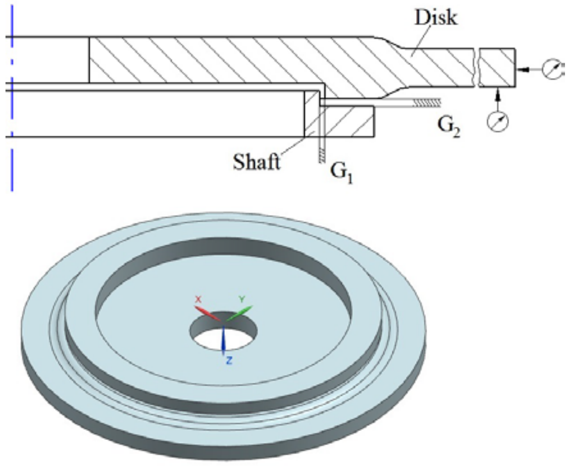

Fig. 1. Assembly configuration

We can determine the clearance function G2 between the flat surfaces and the clearance function G1 between the cylindrical surfaces. Cylindrical surfaces are designed to align disc axis relative to the shaft axis. The flat surfaces are required for fixing the axial position of disc.

The mating cylindrical surfaces have a diameter ρ_{cyl} equal to 200 mm. The boundaries of the mating flat surfaces are circles with a radius $\rho_{pl.}$ equal to 235mm. Two surfaces are provided on the disc surface for determining the axial and radial runouts. The radius of the disc surface ρ_{disk} for determining the mechanical and radial runouts is 300mm. The height H of the cylindrical surfaces of the shaft and the disc is 10mm.

The coordinates of the cylindrical surfaces are calculated by the equations

$$\begin{cases} x = \rho_{cyl} \cdot \cos t, \\ y = \rho_{cyl} \cdot \sin t, \\ \quad z = h, \end{cases} \tag{5}$$

where t is the angle of the cylindrical surface section, which varies in the range of 0 to 360 degrees;

h is the height of the cross section along the z –axis and it changes from 0 to N.

For flat surfaces of the shaft flange and disc the coordinates of the points are calculated by the following equations:

$$\begin{cases} x = \rho \cdot cos\, t, \\ y = \rho \cdot sin\, t, \\ z = const, \end{cases} \quad (6)$$

Where ρ is sectional radius which varies in the range $[\rho_{cyl.}; \rho_{pl.}]$ for the shaft and in the range $[\rho_{cyl.}; \rho_{disk}]$ for the disc.

Cylindrical surface are determined at 360 evenly distributed points in two sections parallel to the XOY plane. The flat surface of the mating shaft is a limited area located between two circles. It is given by three arrays of points arranged in a circle with a radius Rcyl, (Rcyl + Rpl) / 2 and Rpl. The flat surface of the disc is also a limited area located between the two circles. It is described by three arrays of pixels arranged in a circle with a radius Rcyl, (Rcyl + Rdisc) / 2 and Rdisc. Each circle points array has 360 points respectively.

3.1 Simulation of deviation of surfaces form

Simulation of form deviations of cylindrical surfaces was carried out by harmonic function and in the case of the flat mating surface of the shaft by a quadratic function. The functions of form deviations, which we used, are performed below:

1) For a cylindrical surface of the shaft in XOY section:

$$dF_{cyl1} = A_1 \cdot (1 - \sin(t)/3) \cdot \cos(k_1 \cdot t + \pi/2) + A_2 \cdot (1 - \cos(t)) \cdot \sin(k_2 \cdot t + \pi/2), \quad (7)$$

Where A_1, A_2 are the amplitudes of harmonics, mm;

k_1, k_2 are harmonic frequencies;

2) For the cylindrical surface of the disc mating with the shaft in the XOY section:

$$dF_{cyl2} = A_1 \cdot (1 - \sin(t)/2) \cdot \cos(k_1 \cdot t + 4) + A_2 \cdot (1 - \cos(t)/4) \cdot \sin(k_2 \cdot t + \pi/2), \quad (8)$$

3) Form deviation of the shaft side plane is described by a quadratic equation. For the i-th section of the side plane the deviation is calculated by the formula:

$$\mathrm{dF}_{pl.} = (-\mathrm{A}_{pl.})/(\rho_{pl.} - \rho_{cyl.})^2 \cdot (\rho_i - \rho_{cyl.})\, t)^2 + \mathrm{A}_{pl.}, \quad (9)$$

ρ_i is the radius of the current section

$A_{pl.}$ is the maximum value of the form deviation of the side plane.

Between the cylindrical surfaces assigned guaranteed clearance s of 10 and 20 microns. Table 1 shows the parameters of imposed deviations.

Table 1. Values of the form deviations of the mating surfaces.

Parameter	A_1, μm	A_2, μm	k_1	k_2	$A_{pl.}$, μm	s, μm
Value	20	12,5	3	6	50	10, 20

3.2 Estimate results of the uncertainty of the parts assembly

To estimate the uncertainty of the assembly parameters of shaft and disc with deviations form, the characteristics of which are summarized in Table 1, parts assembly simulation was produced multiple times using conjugation model discussed in Section 2.1. Assembly options were differ in the direction of the assembly effort vector $\overline{D_1}$ and in the initial angular position α of the disc relative to the shaft by the rotation of the first plane XOY at a pitch of 9º. The value of the vector $\overline{D_1}$ and the angular position α of the disc are shown in Table 2.

Table 2. Parameters of parts assembly conditions

Parameter	Values
$\overline{D_1}$	(0,0,1); (0.44,0 0.89);(0.7,0,0.7); (0.89,0,0.44); (1,0,0)
α	0º -351º

Thus the calculation is made for 200 conjugations with two clearance values.

To estimate the influence of form deviation on assembling accuracy we used following assembly parameters: ρ_{centr} radial runout of the cylindrical surface of the disc; Δz displacement of an end plane of the disc relative to the shaft plane along Z axis; F_r face runout of the disc relative to shaft end calculated as the difference between the maximum and minimum values of the disc coordinates of points in the section with radius of 300mm along Z axis. For each realization the ξ_a calculation error of conjugation was calculated, it is equal to the value of the gap function.

Histograms of the distribution of spatial conjugation uncertainties are shown in Fig. 2.

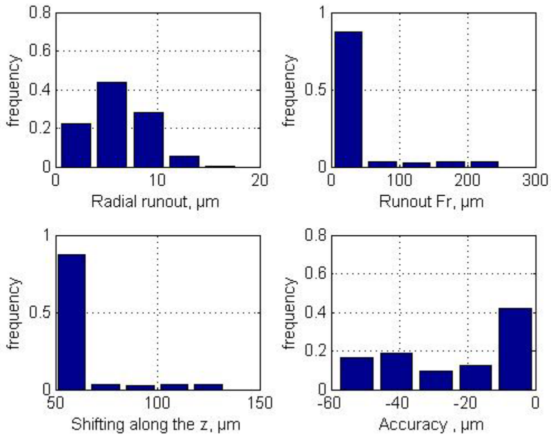

Fig. 2. Histograms of the distribution of spatial conjugation uncertainties

An expectation and a root-mean-square deviation were calculated for each considered uncertainty parameter, as well as for conjugation calculation error. The results are shown in table 3.

Table 3. Error parameters for conjugation of the shaft with disc.

Parameter	Clearance 10 µm		Clearance µm	
	M	RMSD	MAX	MIN
ρ_{centr} , µm	6,14	2,98	12,87	6,02
F_r , µm	39,34	41,78	48,86	45,14
Δz , µm	63,67	13,97	66,66	14,97
ξ_a , µm	-27,30	9,73	-4,40	2,14

Analysis of the results revealed that the ξ_a value is relatively low when the value of the clearance equal to the form deviation. However, if the clearance is smaller than the form deviation the value in modulus increases significantly. The appearance of a considerable value of ξ_a indicates occurrence of interference in the shaft-hole assembly. An expectation and a root-mean-square deviation of the ρ_{centr} parameter increased approximately 2-fold as clearance increased 2-fold. Face runout, which is characterized by the difference between the largest and smallest distances from the end disc surface points to the plane perpendicular to the Z axis of disc rotation, reaches a significant value in both cases. Face runout in this case depends on the clearance between the cylindrical surfaces and form deviation of the shaft end surface wherefore disc takes up the sag about the Z axis. The minimum value of Δz is the form deviation of end surface. Δz displacement along the axis of rotation reaches a maximum when force vector (1,0,0) is applying, i.e. in the plane perpendicular to the axis of rotation.

4. CONCLUSION

The paper describes the estimation model of uncertainties of spatial mating of high-precision parts. The model can be used to create methodologies for calculating dimensional chains with a view to taking into account the form errors of mating surfaces of the parts. The development of such methods will allow evaluating the effect of geometrical deviations of their elements in the assembly accuracy. Implementation of the model is shown on the example of shaft and disc assembly. The assembly is made on cylindrical and flat surfaces. Form precision of cylindrical surfaces is 0.02 mm and 0.05 mm for flat surfaces. A clearance which equals 0.01 mm and 0.02 mm is supported between the cylindrical surfaces. Mating uncertainties were determined for the geometric parameters of coordinates of the disc axis center.

The results of conducted research led to the conclusion about the significant influence of the form deviation of the mating surfaces on the uncertainty of their conjugation. Form deviations of manufactured parts have a significant impact on their relative position. Use of estimates of the uncertainty for calculating complex assemblies with complex form deviations makes possible to evaluate manufacturing tolerances and to solve task of tolerances assignment.

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